Announcements:

- Webwork is posted, due Monday 10pm
- HW3B (only) is due on Friday
- Midterm:
  - Section 102 (9am) average: 22
  - Section 103 (1pm) average: 28
  Probably, section 102 had a harder test. We will likely have to re-scale the term marks a little, based on the relative final exam performances of the sections.

This week’s topics:

- Second order constant-coefficient ODE
  - Unforced (homogeneous) and forced
- Applications of second order
- Natural frequency
- Resonance

- Reading: Lebl 2.1, 2.2, 2.4, 2.5, 2.6

A branch sways back and forth with position $f(t)$. Studying its motion you find that its acceleration is proportional to its position, so that when it is 8 cm to the right, it will accelerate to the left at a rate of 2 cm/s². Which differential equation describes the motion of the branch?

(a) $\frac{d^2 f}{dt^2} = 8f$
(b) $\frac{d^2 f}{dt^2} = -4f$
(c) $\frac{d^2 f}{dt^2} = -2$
(d) $\frac{d^2 f}{dt^2} = \frac{f}{4}$
(e) $\frac{d^2 f}{dt^2} = -\frac{f}{4}$

Test the following functions to see which is a solution to $y'' + 4y' + 3y = 0$.

(a) $y = e^{2t}$
(b) $y = e^t$
(c) $y = e^{-t}$
(d) $y = e^{-2t}$
(e) None of these are solutions.
Test the following functions to see which is a solution to \( \frac{d^2 y}{dt^2} + 2 \frac{dy}{dt} + 2y = 0 \).

(a) \( g = e^x \)
(b) \( g = \sin x \)
(c) \( g = e^{-x} \sin x \)
(d) None of these are solutions.

Suppose we want to solve the differential equation \( y'' + ay' + by = 0 \) and we conjecture that our solution is of the form \( y = Ce^{rt} \). What equation do we get if we test this solution and simplify the result?

(a) \( 1 + ar + b r^2 = 0 \)
(b) \( C^2 r^2 + C r + c = 0 \)
(c) \( C e^{rt} + aC e^{rt} + bC e^{rt} = 0 \)
(d) \( r^2 + ar + b = 0 \)
(e) None of the above

Find the general solution to \( \frac{d^2 y}{dt^2} + 3 \frac{dy}{dt} + 2y = 0 \).

(a) \( y(t) = C_1 e^{-t/2} + C_2 e^{t/2} \)
(b) \( y(t) = C_1 e^{-2t} + C_2 e^{-t} \)
(c) \( y(t) = C_1 e^{-2t} + C_2 e^t \)
(d) \( y(t) = -2C_1 e^{-2t} - C_2 e^{-t} \)
(e) None of the above
Homogeneous (unforced) \( y'' + by' + cy = 0 \)
forced \( y'' + by' + cy = g(t) \)
b, c are constants. These are linear equations.

Propaganda

1. Spring-mass system

\[
\begin{align*}
\text{Hooke's law: spring force is proportional to displacement.} \\
x' &= 0 \text{ equilibrium}
\end{align*}
\]

N2L

\[
F = ma \\
-kx = m \frac{d^2x}{dt^2}
\]

undamped harmonic motion
build in a friction term proportional to velocity,
\[ x'' + \sigma x' + \frac{k}{m} x = 0 \]
damped equation
friction coefficient \( \sigma > 0 \)

2) LCR circuit

Kirchhoff 2nd law sum of voltage drops across = applied voltage components

Define \( Q(t) = \) charge on capacitor \( I(t) = \) current

\[ I = \frac{dQ}{dt} \]

\[ \frac{1}{C} Q + L \int \frac{dI}{dt} + IR = V(t) \]

\[ L \frac{d^2Q}{dt^2} + R \frac{dQ}{dt} + \frac{1}{C} Q = V(t) \]
\[ Q'' + \frac{R}{L} Q' + \frac{1}{cL} Q = \frac{1}{L} v(t) \]  

Pendulum

Include damping

\[ mL\theta'' = -mg\sin\theta - mgc \frac{d\theta}{dt} \]

Nonlinear Pendulum (with damping)

Approximate \( \sin\theta \approx \theta \) (valid for small \( \theta \)) (small-angle approximation)

Damped (linear) pendulum

Undamped case

\[ \theta'' + \frac{g}{L} \theta = 0 \]

\[ \theta'' + \frac{g}{L} \theta = 0 \]