Math 215, Winter 2014
Midterm 3, March 27

Name: SID:
Instructor: Section:

Instructions
• The total time allowed is 60 minutes.
• The total score is 50 points.
• Use the reverse side of each page if you need extra space.
• Show all your work. A correct answer without intermediate steps will receive no credit.
• Calculators, phones and cheat sheets are not allowed.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Points</th>
<th>Score</th>
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<tbody>
<tr>
<td>1</td>
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<td>TOTAL</td>
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1. (16 marks)
   a) The general solution to \( \frac{dx(t)}{dt} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} x(t) \) is
   \[
   x(t) = c_1 \begin{pmatrix} 1 \\ -1 \end{pmatrix} + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{2t}.
   \]
   Find the general solution to
   \[
   \frac{dx(t)}{dt} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} x(t) + \begin{pmatrix} 1 \\ 0 \end{pmatrix}.
   \]
2. **Matching** (12 marks: 1 mark each)

In each of the parts *i*-iv below, you are given three constant coefficient linear differential equations with *unspecified* initial conditions. In each grouping, a differential equation matches to one of the options provided.

**Solution (circle the correct answer in each case):**

(i) \( y'' + 4y = 0 \) matches plot \( A \ B \ C \)
    \( y'' + y' + 4y = 0 \) matches plot \( A \ B \ C \)
    \( y'' + 9y = 0 \) matches plot \( A \ B \ C \)

(ii) \( y'' + y' + y = 0 \) matches plot \( D \ E \ F \)
     \( y'' + y' + 2y = 0 \) matches plot \( D \ E \ F \)
     \( y'' + y' + \frac{1}{4}y = 0 \) matches plot \( D \ E \ F \)

(iii) \( y'' + 4y' + 8y = 0 \) matches plot \( G \ H \ I \)
     \( 3y'' + 4y' + 8y = 0 \) matches plot \( G \ H \ I \)
     \( 3y'' + 4y' + \frac{8}{3}y = 0 \) matches plot \( G \ H \ I \)

(iv) \( y'' + y = \sin t \) matches plot \( J \ K \ L \)
     \( y' + y' + 4y = \sin t \) matches plot \( J \ K \ L \)
     \( y'' + 4y = \sin t \) matches plot \( J \ K \ L \)
4. A 1-kg mass on a spring undergoes undamped, free horizontal motion along the $x-$axis with an oscillation period of 2 s. Let $x(t)$ be the displacement of the mass relative to the equilibrium position $x = 0$. At $t = 3$ s, the mass is at position $x = 1$ m with velocity $dx/dt = 4$ m/s.

a. Write down a differential equation for the position $x(t)$. Your equation should include an unknown spring constant $k$.

b. Solve for $x(t)$ in terms of $k$.

c. Determine $k$.

d. What is the amplitude of $x(t)$?