No calculators or notes permitted. This exam has 5 questions. Answer all questions. For each part of each question, draw a box around your final answer.

1. **(25 pts)** Match each direction field to a differential equation, or state that it does not match any of the equations.

   \[
   \begin{align*}
   (1) \quad & \mathbf{x}' = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \mathbf{x} \\
   (2) \quad & \mathbf{x}' = \begin{pmatrix} 1 & 1 \\ -2 & 3 \end{pmatrix} \mathbf{x} \\
   (3) \quad & \mathbf{x}' = \begin{pmatrix} 0 & 1 \\ -2 & 0 \end{pmatrix} \mathbf{x} \\
   (4) \quad & \mathbf{x}' = \begin{pmatrix} -1 & 1 \\ -2 & 3 \end{pmatrix} \mathbf{x} \\
   (5) \quad & \mathbf{x}' = \begin{pmatrix} -3 & 1 \\ -2 & -3 \end{pmatrix} \mathbf{x}
   \end{align*}
   \]
2. (25 pts) Solve the following system of differential equations with initial conditions and sketch the solution. On your sketch, label the initial point, initial velocity vector and asymptotic direction.

\[
x' = \begin{pmatrix} 1 & -4 \\ 4 & -7 \end{pmatrix} x \\
x(0) = \begin{pmatrix} -1 \\ 1 \end{pmatrix}
\]
3. (25 pts) We consider a system of differential equations containing the parameter $a$.

$$x' = \begin{pmatrix} 0 & -5 \\ 1 & a \end{pmatrix} x$$

(a) Determine the eigenvalues in terms of $a$.
(b) Find the critical value(s) of $a$ where the qualitative nature of the solution changes.
(c) Summarize your findings in words.
4. (10pts) Consider the second order homogeneous equation

\[ y''(t) + by'(t) + cy(t) = 0 \]

where \( a, b \) are real constants. This equation is equivalent to a system of two differential equations. Find that system.
5. (15pts) Show that all solutions of the system

\[ x' = \begin{pmatrix} a & b \\ c & d \end{pmatrix} x \]

approach zero as \( t \to \infty \) if and only if \( a + d < 0 \) and \( ad - bc > 0 \).