

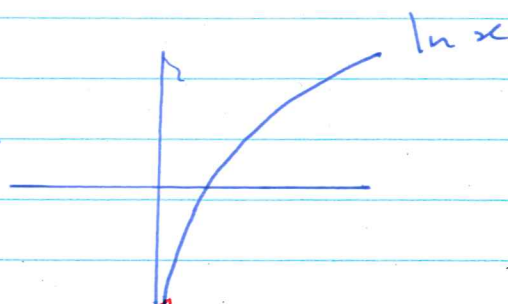
①

Oct 5

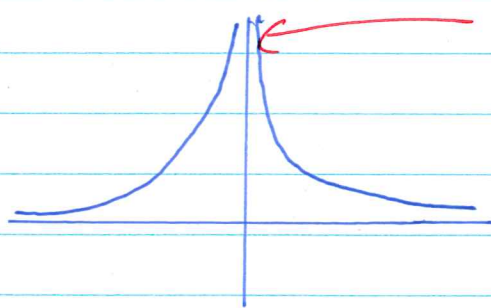
- HW # 3 is due.
- Quiz # 2 Friday
 - limits (lectures last week)
 - Week 4 practice problems
 - vertical but no horizontal asymptotes
- HW # 4 Due ~~Monday~~ ^{asymptotes} Wednesday.

Asymptotes (§ 3.6)

- $\lim_{x \rightarrow 0^+} \ln x = -\infty$



- $\lim_{x \rightarrow 0} \frac{1}{x^2} = \infty$



vertical asymptotes at $x = 0$

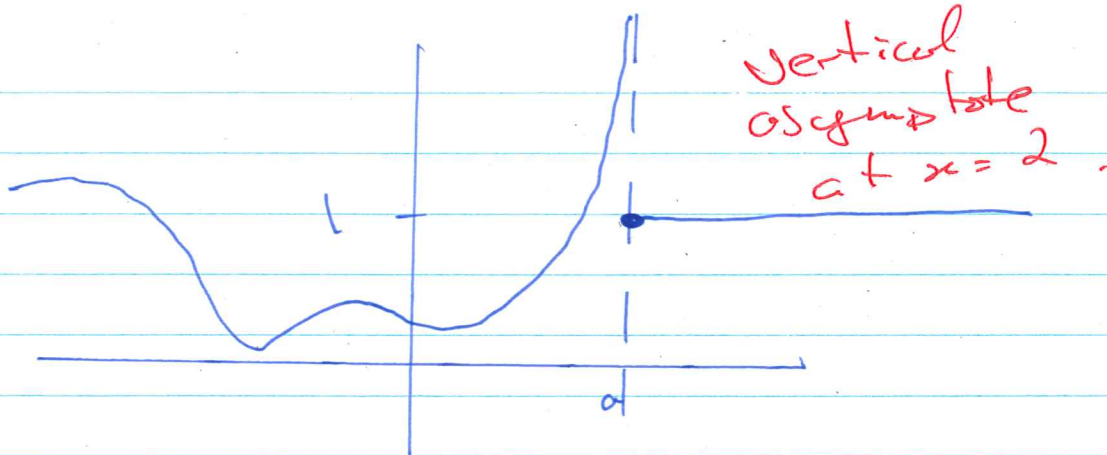
What does it mean for a function to have a vertical asymptote at $x = a$

That either (or both)

- $\lim_{x \rightarrow a^-} f(x) = \infty$ (or $-\infty$)

- $\lim_{x \rightarrow a^+} f(x) = \infty$ (or $-\infty$)

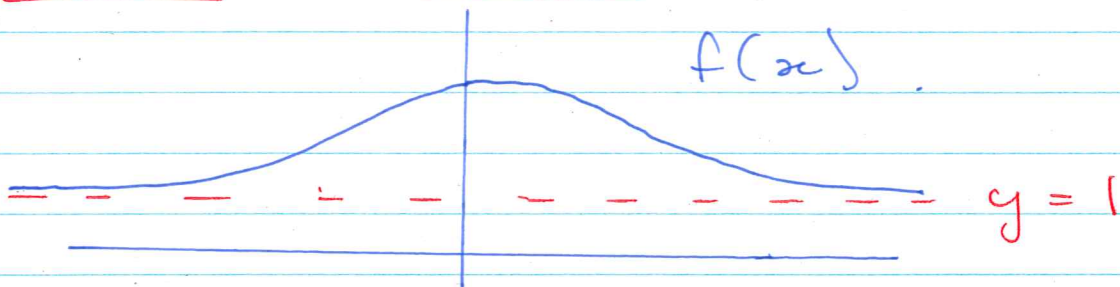
②



$$\lim_{x \rightarrow a^-} f(x) = \infty$$

Even though $f(a) = 1$.
Still V.A. at $x = a$.

Horizontal Asymptotes



What does it mean for a function to have a horizontal asymptote?

We write $\lim_{x \rightarrow \infty} f(x) = 1$.

$\lim_{x \rightarrow -\infty} f(x) = 1$.

③

As x gets large, $f(x)$ gets close to 1.

In general,

$$\text{If } \lim_{x \rightarrow \infty} f(x) = L$$

$$\text{AND/OR } \lim_{x \rightarrow -\infty} f(x) = L.$$

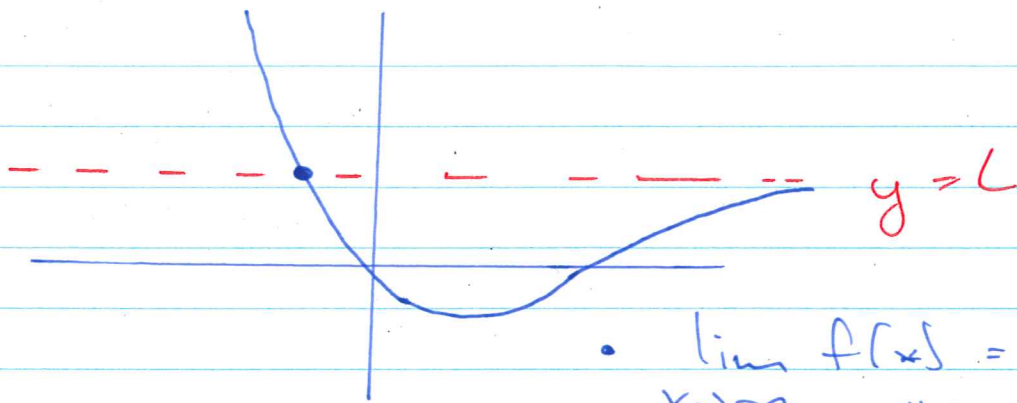
We say $f(x)$ has a horizontal asymptote at $y = L$.

Clicker Q: Can a function cross its horizontal asymptote?

- A) Yes
B) No

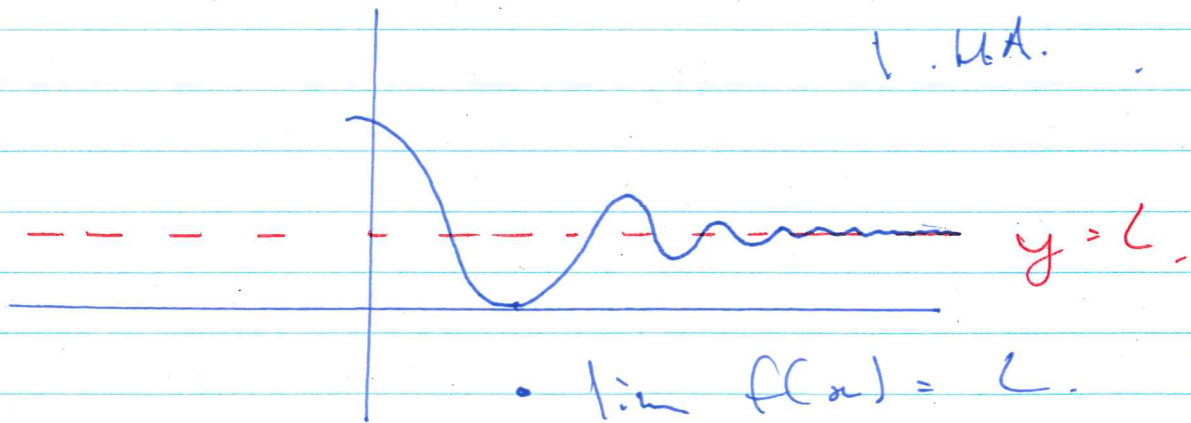
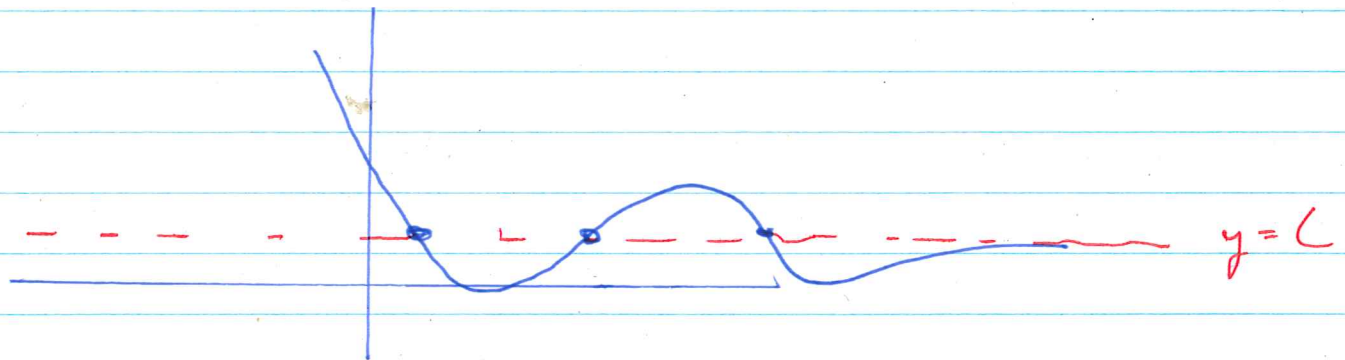
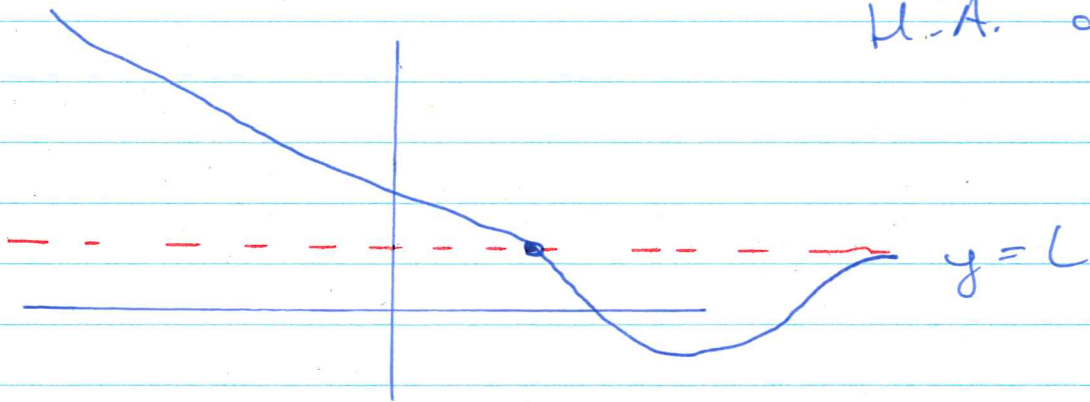


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• $\lim_{x \rightarrow \infty} f(x) = L.$

(H)
H.A. at $y=L.$



• $\lim_{x \rightarrow \infty} f(x) = L.$

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Clicker Q: What is the max number of H.A. a function can have?

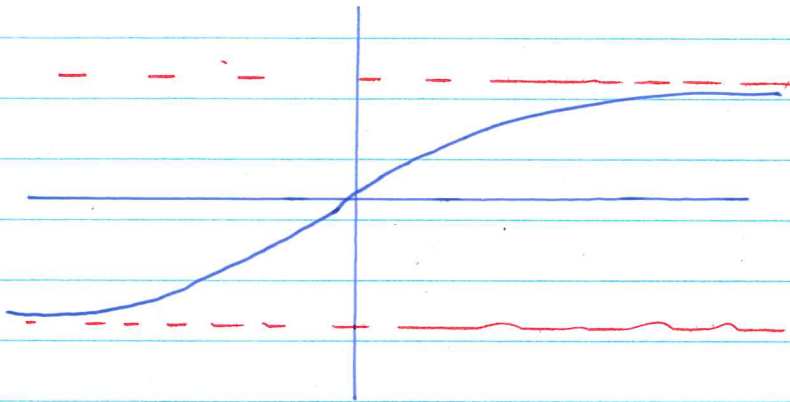
A) 0

B) 1

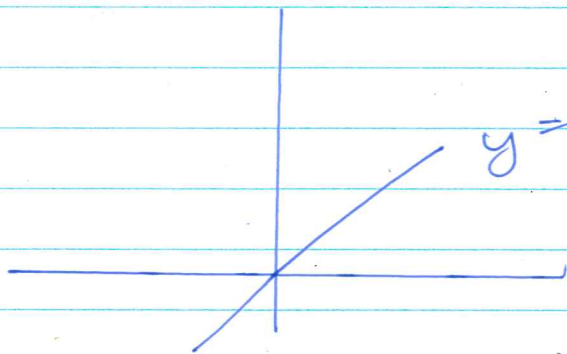
→ C) 2

D) 3

E) Infinitely many

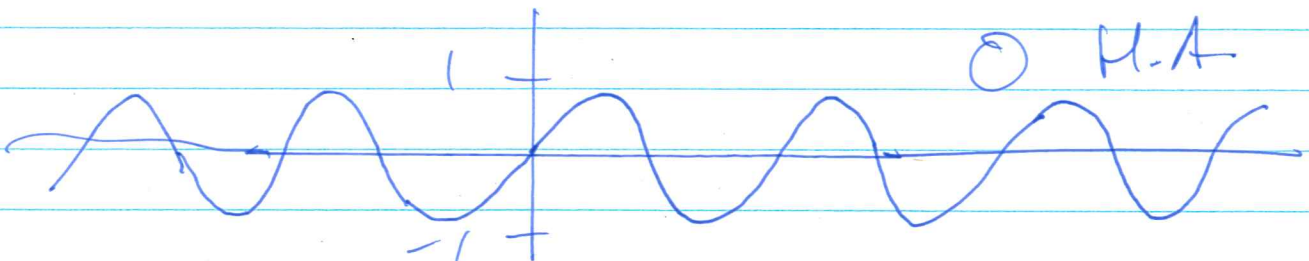


2 H.A.



0 H.A.

Does the Sine function have a H.A.?



No H.A. $\Leftrightarrow \left\{ \begin{array}{l} \lim_{x \rightarrow \infty} \sin x \text{ D.N.E.} \\ \lim_{x \rightarrow -\infty} \sin x \text{ D.N.E.} \end{array} \right.$

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Example: Find all H.A. of

$$f(x) = \frac{2x^3 + x^2 + 4}{x^3 - 7x + 1}$$

Compute

$$\lim_{x \rightarrow \infty} \frac{2x^3 + x^2 + 4}{x^3 - 7x + 1}$$

Divide each term by the highest power.

$$= \lim_{x \rightarrow \infty} \frac{2x^3/x^3 + x^2/x^3 + 4/x^3}{x^3/x^3 - 7x/x^3 + 1/x^3}$$

$$= \lim_{x \rightarrow \infty} \frac{2 + \frac{1}{x} + \frac{4}{x^3}}{1 - \frac{7}{x^2} + \frac{1}{x^3}}$$

all go to zero as $x \rightarrow \infty$.

$$= \frac{2 + 0 + 0}{1 - 0 + 0} = 2$$

\Rightarrow H.A. at $y = 2$.

Similarly, $\lim_{x \rightarrow -\infty} f(x) = 2$.