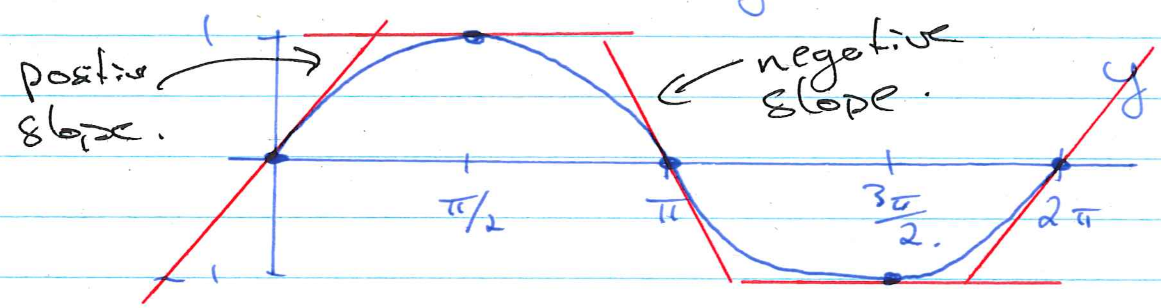
Oct. 16.

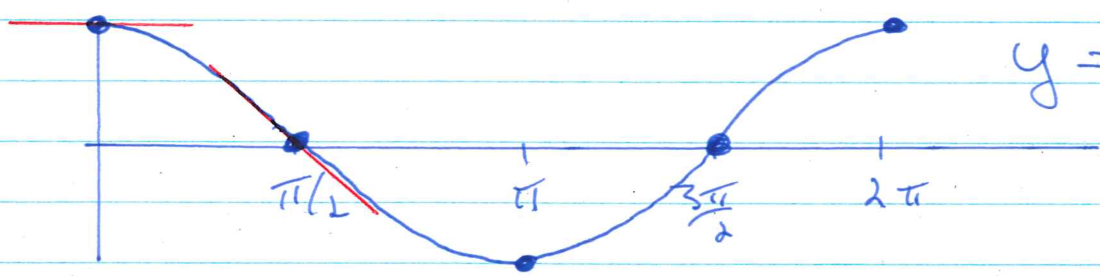
- HW4 Solutions posted
- HW5 Due Monday
- Quiz #3 Oct. 23 Friday.
- Midterm Nov. 2
- Learning Objectives / Practice Problems updated
- Grades posted.

Last class: derivatives of polynomials and exponential functions.

What about trig. functions?



Let's try to sketch the derivative.



We think $\frac{d}{dx} \sin x = \cos x$.

This can be proved using the limit definition:

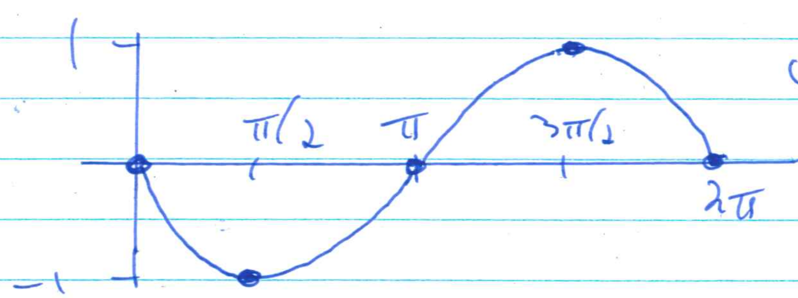
$$\frac{d}{dx}(\sin x) = \lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h}$$

Lots of trig $\left\{ \begin{array}{l} = \quad \vdots \end{array} \right.$

$$= \cos x$$

By the way, $(\sin x)' = \cos x$ only if we use radians.

We can also sketch out the derivative of $\cos x$:



$y = -\sin x$.

$$\left\{ \begin{array}{l} \dfrac{d}{dx}(\sin x) = \cos x \\[2ex] \dfrac{d}{dx}(\cos x) = -\sin x \end{array} \right.$$

Clicker Q: What is the derivative of $f(x) = 3\sin x$?

A) 3 → C) $3\cos x$
B) 0 D) $\cos x$

constant.

Note: $\frac{d}{dx}\left(c\, f(x)\right) = c \cdot \frac{df}{dx}$.

Recall! $(f+g)' = f' + g'$
$(f-g)' = f' - g'$.

Clicker Q: Is $(x\sin x)' = 1 \cdot \cos x$?

A) Yes
→ B) No
C) I don't know because we haven't learned product rule yet.

④

Easier Example: $h(x) = (x+1)(x-2)$.
Find $h'(x)$.

We could expand:

$$h(x) = x^2 - x - 2$$
$$h'(x) = 2x - 1 + 0$$

So can we take the derivative of both terms and multiply?

$$h'(x) \cancel{=} 1 \cdot 1$$

We need Product Rule:

$$(f \cdot g)' = f'g + fg'$$

$$h(x) = f(x)\,g(x), \quad f(x) = x+1$$
$$g(x) = x-2$$
$$f'(x) = 1$$
$$g'(x) = 1$$

$$h'(x) = f'g + fg'$$
$$= 1 \cdot (x-2) + (x+1) \cdot 1$$

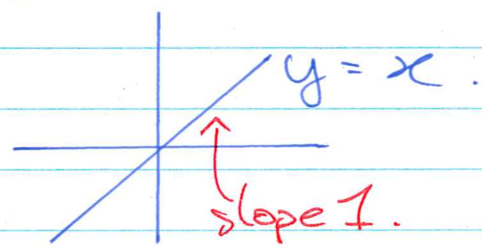
Rule: $h'(x) = f'g + fg'$.

$$h(x) = \underbrace{(x+1)}_{f(x)}\underbrace{(x-2)}_{g(x)}.$$

$f(x) = x+1$, $f'(x) = 1$.

$g(x) = x-2$, $g'(x) = 1$.

(aside: $f(x) = x$



power rule

$f(x) = x^1$

$f'(x) = 1x^{1-1}$

$= 1x^0$

$= 1$.)

$$h'(x) = 1\cdot(x-2) + (x+1)\cdot 1.$$

$$= x - 2 + x + 1$$

$$= \underline{2x - 1} \leftarrow \text{as expected!}$$

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Find the derivative of each

Examples:

1) $x \sin x$
2) $7 \sin x \cos x$.
3) xe^{x}

Clickers:

A) 1 done
B) 2 done
C) 3 done.

1). $h(x) = \underbrace{x}_{f(x)} \cdot \underbrace{\sin x}_{g(x)}$.

$f(x) = x$ $\qquad$ $g(x) = \sin x$

$f'(x) = 1$ $\qquad$ $g'(x) = \cos x$.

$$h'(x) = 1 \cdot \sin x + x \cos x$$

$$= \sin x + x \cos x.$$

2) $h(x) = \underbrace{7\sin x}_{f(x)} \cdot \underbrace{\cos x}_{g(x)}.$

$f(x) = 7\sin x \qquad f'(x) = 7\cos x$

$g(x) = \cos x. \qquad g'(x) = -\sin x.$

$h' = f' \cdot g + f \cdot g'$

$= 7\cos x \cos x + 7\sin x(-\sin x)$

Simplification is not necessary

$= 7\cos^2 x - 7\sin^2 x.$

$= 7(\cos^2 x - \sin^2 x).$

3) $h(x) = xe^x.$

$f(x) = x \qquad f'(x) = 1$

$g(x) = e^x. \qquad g'(x) = e^x.$

$h'(x) = f'g + fg'$

$= 1 \cdot e^x + xe^x.$

$= e^x + xe^x$

(aside: why is $(e^x)' = e^x$.

Need limit definition.

$$\lim_{h \to 0} \frac{e^{x+h} - e^x}{h} = \ldots = e^x.$$)

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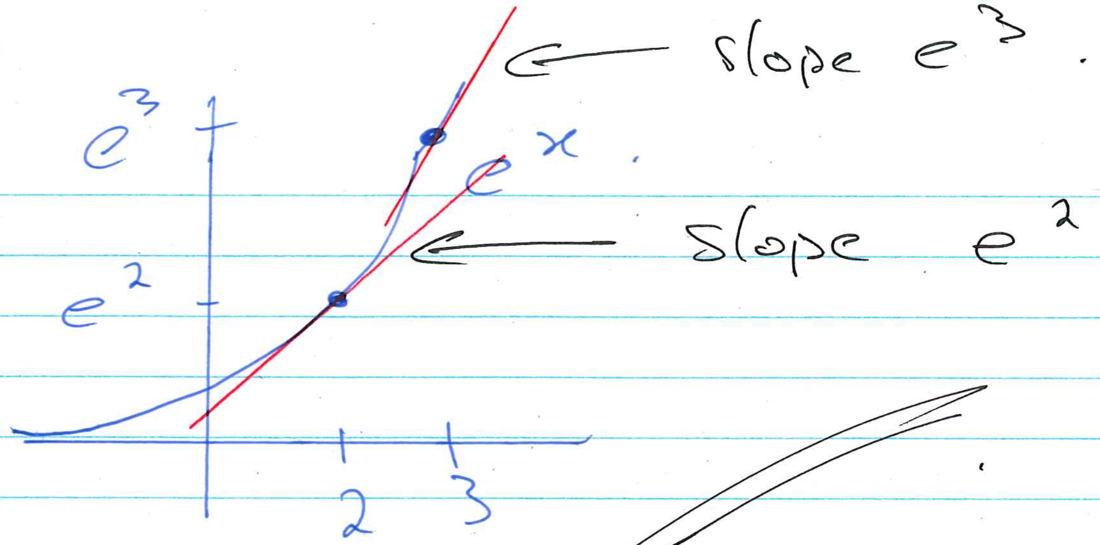

$$(a^x) = a^x \ln a$$

$$(e^x) = e^x \ln e = e^x.$$