Before you look at these solutions ask yourself:

Did I try the problem for 10 minutes?

If not, try again.

②

M190: Final Exam Practice

1. • $\left(\frac{e^{\sin x}}{\ln x}\right)' = \frac{\cos x\, e^{\sin x} \ln x - \frac{1}{x} e^{\sin x}}{(\ln x)^2}$.

• $\left(\sqrt{x^2+\cos x}\right)' = \frac{1}{2}\left(x^2+\cos x\right)^{-1/2} \cdot \left(2x - \sin x\right)$.

• $\left(x^2 e^{1/x}\right)' = 2x e^{1/x} + x^2 e^{1/x}\left(\frac{-1}{x^2}\right)$.

2. $f(x) = \ln x \cos(\pi x)$.

$f'(x) = \frac{1}{x}\cos(\pi x) - \pi \ln x \sin(\pi x)$

$f'(1) = \frac{1}{1}\cos(\pi) - \pi \ln(1) \sin(\pi)$

$= -1$.

$y - y_0 = m(x - x_0)$

$y = -1(x-1)$.

$m = -1$

$x_0 = 1$

$y_0 = f(1) = \ln(1)\cos(\pi)$

$= 0$.

3. $f(x) = x^3 - 3x^2 - 9x + 14$

$f'(x) = 3x^2 - 6x - 9$

Want $f'(x) = 0 = 3x^2 - 6x - 9$

$0 = 3(x^2 - 2x - 3)$

$0 = 3(x+1)(x-3)$

So, $x = -1, 3$.

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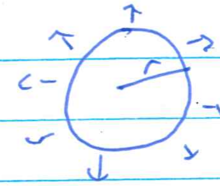
4. $\frac{dr}{dt} = 3$ cm/s

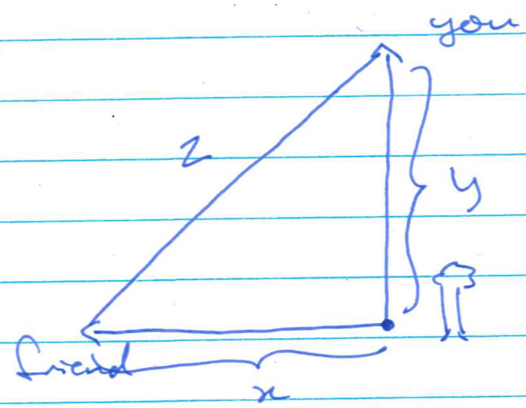


a) $\frac{dC}{dt} = ?$ $\qquad \frac{dC}{dt} = 2\pi \frac{dr}{dt}$

Chain Rule $\qquad = 2\pi \cdot 3$ cm/s.

b) $\frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt} = 2\pi(10) \cdot 3$ cm²/s.

5.



Want: $\frac{dz}{dt}$ $\qquad$ have: $\frac{dy}{dt} = 90$, $\frac{dx}{dt} = 100$.

when $x = 21$, $y = 20$.

Eq. $\qquad z^2 = x^2 + y^2 \quad \rightarrow \quad z^2 = (21)^2 + (20)^2$

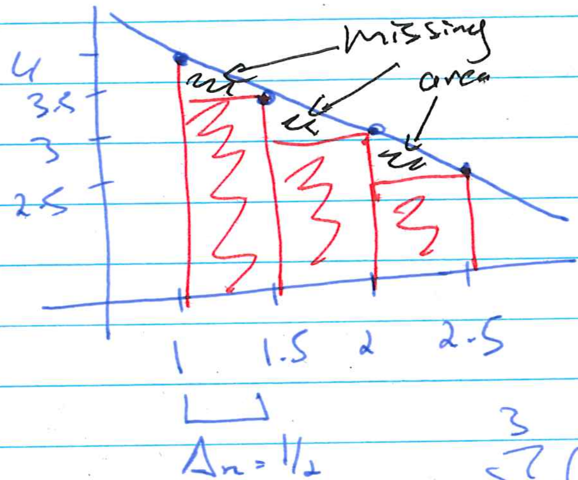
$z = 29$.

Chain Rule: $\cancel{2}z\frac{dz}{dt} = \cancel{2}x\frac{dx}{dt} + \cancel{2}y\frac{dy}{dt}$

$$\frac{dz}{dt} = \frac{1}{z}\left(x\frac{dx}{dt} + y\frac{dy}{dt}\right)$$

$$= \frac{1}{29}\left(21 \cdot 100 + 20 \cdot 90\right) \text{ km/h.}$$

6. $\displaystyle\int_1^{5/2} (5-x)\,dx$



$$\sum_{i=1}^{3} f(x_i)\Delta x$$

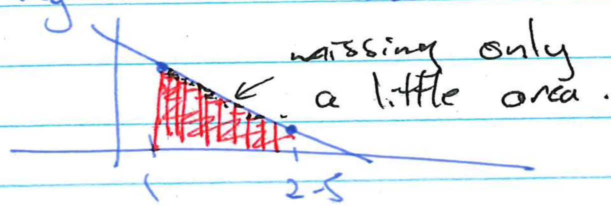
$x_1 = 1.5 \quad f(x_1) = 3.5$
$x_2 = 2 \quad f(x_2) = 3$
$x_3 = 2.5 \quad f(x_3) = 2.5$

$$\sum_{i=1}^{3} f(x_i)\Delta x = \frac{1}{2}\cdot 3.5 + \frac{1}{2}\cdot 3 + \frac{1}{2}\cdot 2.5$$
$$= \frac{1}{2}(3.5+3+2.5)$$
$$= \frac{1}{2}\cdot 6 = 3$$

~~Over~~ Underestimate.
Observe from the picture that we have missed some area.

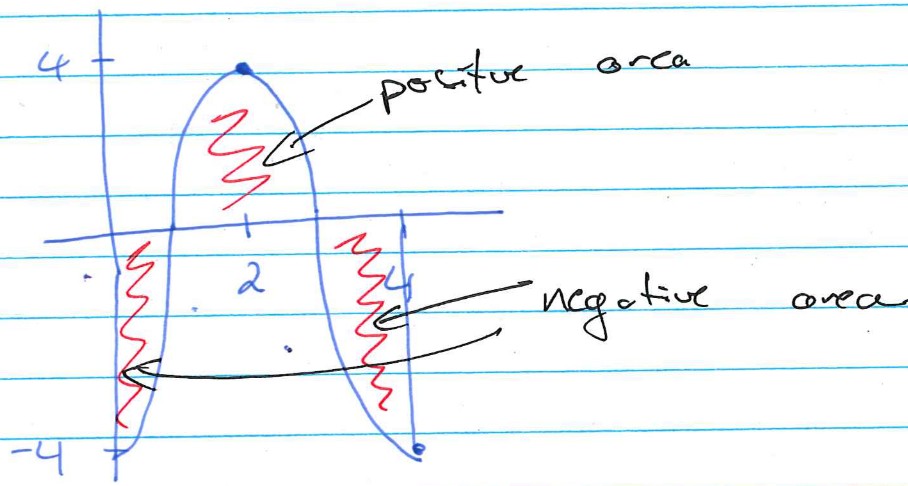
Check: $\displaystyle\int_1^{5/2}(5-x)\,dx = 5x - \frac{x^2}{2}\Big|_{x=1}^{x=5/2}$
$$= 5\cdot\frac{5}{2} - \frac{1}{2}\left(\frac{5}{2}\right)^2 - 5 + \frac{1}{2}$$
$$= 4.875 > 3.$$

The approximation can be made better by increasing the number of bars (n).



If we take $\lim_{n\to\infty}$ then we will achieve the area exactly.

7.



8. $\displaystyle \bullet \int_1^2 \left(\sqrt{x} - \frac{3}{\sqrt{x}} + 5x^2\right) dx$

$$= \frac{2x^{3/2}}{3} - 3\cdot 2(x)^{1/2} + \frac{5}{3}x^3 \Bigg|_{x=1}^{x=2}$$

$$= \frac{2}{3}2^{3/2} - 6\cdot 2^{1/2} + \frac{5}{3}(2)^3$$

$$- 2/3 + 3\cdot 2 - 5/3.$$

$$\bullet \int_{\ln 2}^{\ln 3} e^{-4x}\, dx = -\frac{e^{-4x}}{4}\Bigg|_{\ln 2}^{\ln 3}$$

or let $\begin{cases} u = -4x. \\ du = -4dx. \end{cases}$

$$= -\frac{1}{4}\left(e^{-4\ln 3} - e^{-4\ln 2}\right)$$

$$= -\frac{1}{4}\left(3^{-4} - 2^{-4}\right)$$

$$\bullet \int_{\pi/2}^{\pi} \cos(3x+\pi)\, dx = \frac{1}{3}\sin(3x+\pi)\Bigg|_{\pi/2}^{\pi}$$

or let $u = 3x + \pi$
$du = 3dx.$

$$= \frac{1}{3}\sin(4\pi) - \frac{1}{3}\sin\left(\frac{5\pi}{2}\right)$$

$$= 1/3 \cdot 0 - \frac{1}{3}\cdot 1 = -1/3.$$

ε

$$\bullet \int_2^5 x e^{-x}\, dx \qquad \text{let} \quad u = x \quad dv = e^{-x} dx$$
$$du = dx \quad v = -e^{-x}$$

$$= -xe^{-x}\Big|_2^5 + \int e^{-x}\, dx$$

$$= -xe^{-x}\Big|_2^5 - e^{-x}\Big|_2^5$$

$$= -5e^{-5} + 2e^{-2} - e^{-5} + e^{-2}$$
$$= -6e^{-5} + 3e^{-2}.$$

9. $f'(x) = x^2 + \sin(\pi x)$

$$f(x) = \frac{x^3}{3} - \frac{1}{\pi}\cos(\pi x) + C.$$

$$f(2) = 1/3 = \frac{8}{3} - \frac{1}{\pi}\cos(2\pi) + C$$

$$C = -\frac{7}{3} + 1/\pi.$$

$$f(x) = \frac{x^3}{3} - \frac{1}{\pi}\cos(\pi x) - \frac{7}{3} + 1/\pi.$$

10.

$$\bullet \int x\cos(3x)dx \qquad \text{Let} \quad dv = \cos(3x)dx$$
$$v = \frac{1}{3}\sin(3x)$$

$$= \frac{x}{3}\sin(3x) - \frac{1}{3}\int \sin(3x)dx$$

$$u = x$$

$$= \frac{x}{3}\sin(3x) + \frac{1}{9}\cos(3x) + C. \qquad du = dx.$$

$$\bullet \int \frac{x}{(4-x^2)^2}dx = -\frac{1}{2}\int \frac{1}{u^2}du = \frac{1}{2}\frac{1}{u} + C$$
$$= \frac{1}{2(4-x^2)} + C.$$

let $u = 4 - x^2$
$du = -2x\, dx$

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- $\int e^x \sin(e^x)\,dx$ let $u = e^x$, $du = e^x\,dx$.

$= \int \sin(u)\,du$

$= -\cos u + C = -\cos(e^x) + C$.

- $\int \sqrt{x}\ln x\,dx$ let $u = \ln x$, $dv = x^{1/2}dx$, $du = \frac{1}{x}dx$, $v = \frac{2x^{3/2}}{3}$.

$= \frac{2}{3}x^{3/2}\ln x - \frac{2}{3}\int x^{3/2}\frac{1}{x}\,dx$

$= \frac{2}{3}x^{3/2}\ln x - \frac{2}{3}\int x^{1/2}\,dx$

$= \frac{2}{3}x^{3/2}\ln x - \frac{2}{3}\cdot\frac{2}{3}x^{3/2} + C$.

- $\int x\sqrt{3x^2+7}\,dx$ let $u = 3x^2+7$, $du = 6x\,dx$.

$= \frac{1}{6}\int \sqrt{u}\,du$

$= \frac{1}{6}\frac{2}{3}u^{3/2} + C = \frac{1}{9}(3x^2+7)^{3/2} + C$.

- $\int \sin^4 x\cos x\,dx$ let $u = \sin x$, $du = \cos x\,dx$

$= \int u^4\,du = \frac{u^5}{5} + C = \frac{\sin^5 x}{5} + C$.

11. $v(t) = 6\cancel{x} - \cancel{x}^2$ should be t's.

$s(t) = \int v(t)\,dt = \int 6t - t^2\,dt = 3t^2 - \frac{t^3}{3} + C$.

$s(1) = 2 = 3 - 1/3 + C \Rightarrow C = -2/3 \Big\} \Rightarrow s(t) = 3t^2 - t^3 - 2/3$

$s(3) = 3\cdot 3^2 - 3^3/3 - 2/3 = 18 - 2/3$.

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12. $r(t) = 1 + \cos(2t)$

$$A(t) = \int 1 + \cos(2t)\, dt$$

$$= t + \frac{1}{2}\sin(2t) + C.$$

$$A(\pi/4) = 10 = \pi/4 + \frac{1}{2}\overbrace{\sin(\pi/2)}^{1} + C$$

$$C = 10 - \pi/4 - 1/2.$$

$$A(t) = t + \frac{1}{2}\sin(2t) + 10 - \pi/4 - 1/2$$

$$A(\pi/2) = \pi/2 + \frac{1}{2}\underbrace{\sin(\pi)}_{0} + 10 - \pi/4 - 1/2.$$

$$= 10 + \pi/4 - 1/2.$$