Sept. 30.

Quiz # 2 - Fri. 7th.
- limits (week 4 material, this week)

Limits: Compute the following limits

- $$\lim_{x \to -5} \frac{\frac{1}{x} + \frac{1}{5}}{x+5}$$

try substitution: "$\frac{0}{0}$" (sub fails).

algebra → cancel → sub.

$$= \lim_{x \to -5} \frac{\frac{5}{5x} + \frac{x}{5x}}{x+5}$$

$$= \lim_{x \to -5} \frac{\frac{5+x}{5x}}{\frac{x+5}{1}} = \lim_{x \to -5} \frac{5+x}{5x} \div \frac{x+5}{1}$$

$$= \lim_{x \to -5} \frac{\cancel{5+x}}{5x\,\cancel{(x+5)}}$$

$$= \lim_{x \to -5} \frac{1}{5x} = \frac{1}{5(-5)} = \frac{-1}{25}$$

- $$\lim_{x \to 4} \frac{\sqrt{x} - 2}{x - 4}. \qquad \left(\frac{"0"}{0} \right).$$

$$= \lim_{x \to 4} \frac{\sqrt{x} - 2}{x - 4} \cdot \frac{\sqrt{x} + 2}{\sqrt{x} + 2}.$$

↑ multiply top and bottom by conjugate.

$$= \lim_{x \to 4} \frac{x + \cancel{2\sqrt{x}} - \cancel{2\sqrt{x}} - 4}{(x-4)(\sqrt{x}+2)}$$

$$= \lim_{x \to 4} \frac{\cancel{(x - 4)}}{\cancel{(x-4)}(\sqrt{x}+2)}.$$

$$= \lim_{x \to 4} \frac{1}{(\sqrt{x}+2)} = \frac{1}{\sqrt{4}+2} = \frac{1}{4}.$$

algebra → cancel → sub.

There are still a few types of limits we have yet to talk about.

Let us investigate:

$$\lim_{x \to 0^+} \frac{1}{x}.$$

If x is a small positive number then $1/x$ is a large positive number.

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We write,

$$\lim_{x \to 0^+} \frac{1}{x} = \infty.$$

to mean: "$\frac{1}{x}$ gets arbitrarily large as x approaches zero from above/right".

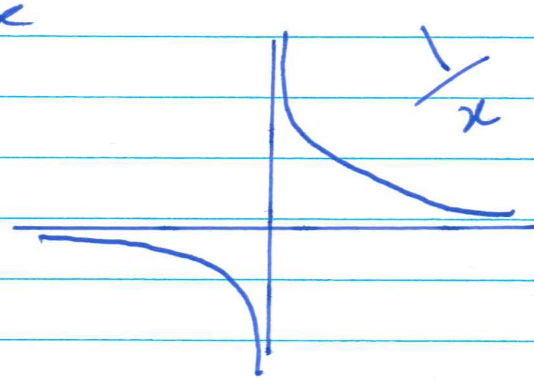
Similarly. $\lim_{x \to 0^-} \frac{1}{x} = -\infty$

means: "$\frac{1}{x}$ gets large (but negative) as $x \to 0^-$".

Note: $\pm\infty$ are not numbers.

Clicker Q: $\lim_{x \to 0} \frac{1}{x} = ?$

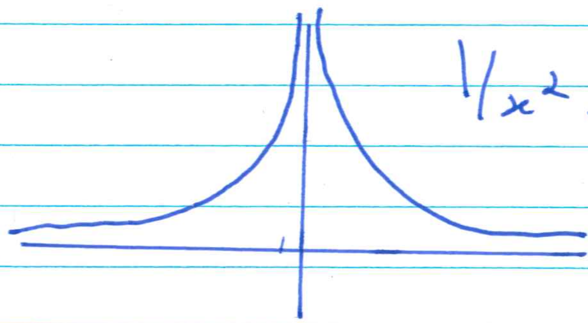
A) ∞
B) $-\infty$
C) $\pm\infty$
D) D.N.E.



Clicker Q: $\lim_{x \to 0} \frac{1}{x^2} = \infty$. (some options).

(A).

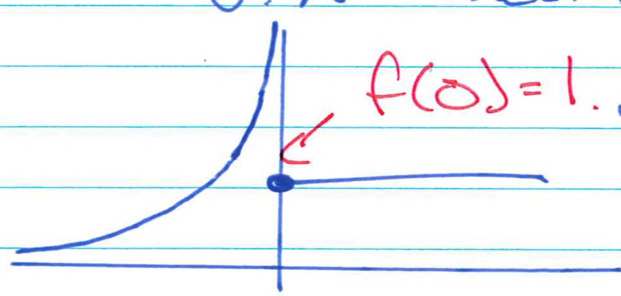
↑ technically this limit D.N.E



Clicker Q: Does this function have a vertical asymptote?

$$f(x) = \begin{cases} \frac{1}{x^2}, & x < 0 \\ 1, & x \geq 0 \end{cases}$$

A) Yes
B) No
C) Don't know.
- because we don't know what V.A. means yet.



(A). Yes! the reason is that

$$\lim_{x \to 0^-} f(x) = \infty .$$

$\Downarrow$ vertical asymptote.

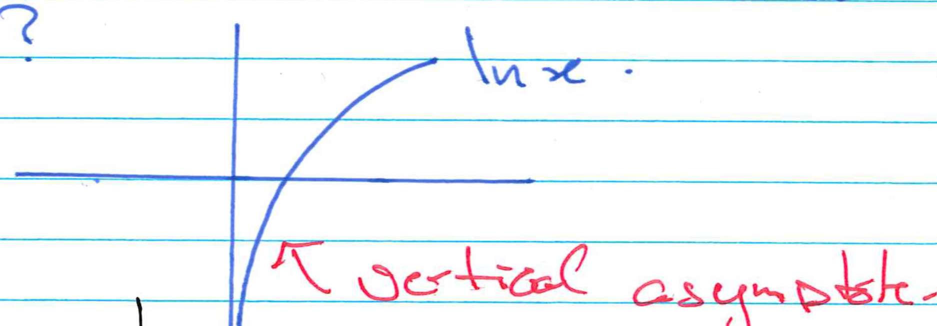
We say a function has a vertical asymptote at $x = a$ if

one of (or both)

- $\lim_{x \to a^-} f(x) = +\infty$ or $-\infty$.
- $\lim_{x \to a^+} f(x) = +\infty$ or $-\infty$.

That is if either of the one sided limits is $+\infty$ or $-\infty$.

Any familiar function with a vertical asymptote?



$$\boxed{\lim_{x \to 0^+} \ln x = -\infty}$$