1

- HW1 returned
  - end of class
  - $\pm\sqrt{1\pm\sqrt{2}}$ $\sqrt{1-\sqrt{2}}$ not a real root.
  - 5 b. - see solutions.
- HW 2 is Due
- HW 3 posted (due Monday)
- Quiz # 2 - Oct. 7.

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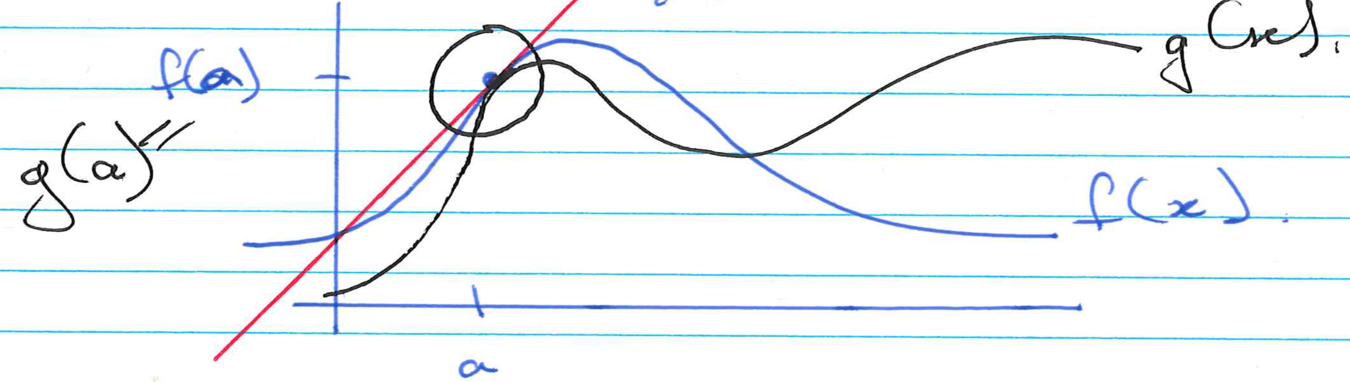



Our goal is to find the slope of the tangent line (speed).

In order to draw the tangent line we need information about $f(x)$ "close" to $x = a$.

We need limits.

Limits: ( Sections 1.1 / 1.2 in Contemporary Calculus )

The limit is an operation that we perform on a function.

We write,

$$\boxed{\lim_{x \to a} f(x) = L}$$

"the limit of $f(x)$ as $x$ approaches $a$ is $L$".

it means: $y = f(x)$ "gets close" to the number $L$ as $x$ "gets closer" to the number $a$.

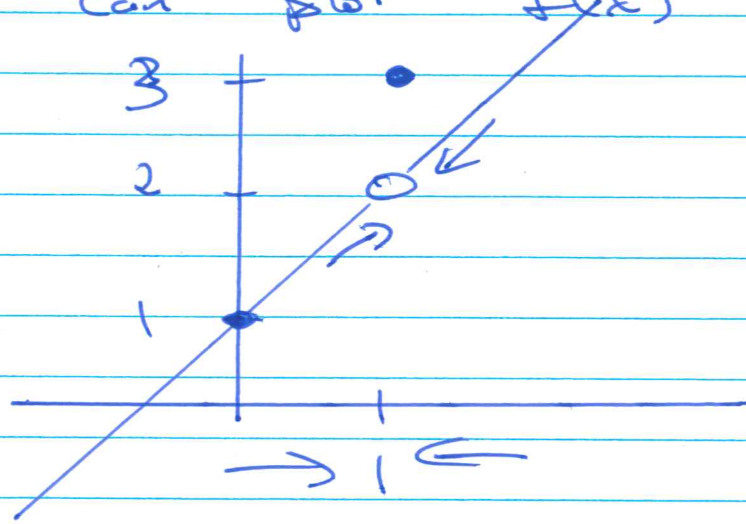
Let's see an example.

Example: Find $\lim_{x \to 1} f(x)$ where

$$f(x) = \begin{cases} x+1 ; & x \neq 1 \\ 3 ; & x = 1 . \end{cases}$$

③

We can plot $f(x)$ to get an idea.



To get an idea of what is going on we can compute some values of $f(x)$ when $x$ is close to $1$.

$f(1.1) = 2.1$ $\qquad$ $f(0.9) = 1.9$

$f(1.01) = 2.01$ $\qquad$ $f(0.99) = 1.99$.

It looks like the values of $f(x)$ are approaching $2$. (this guess is correct.)

We write,

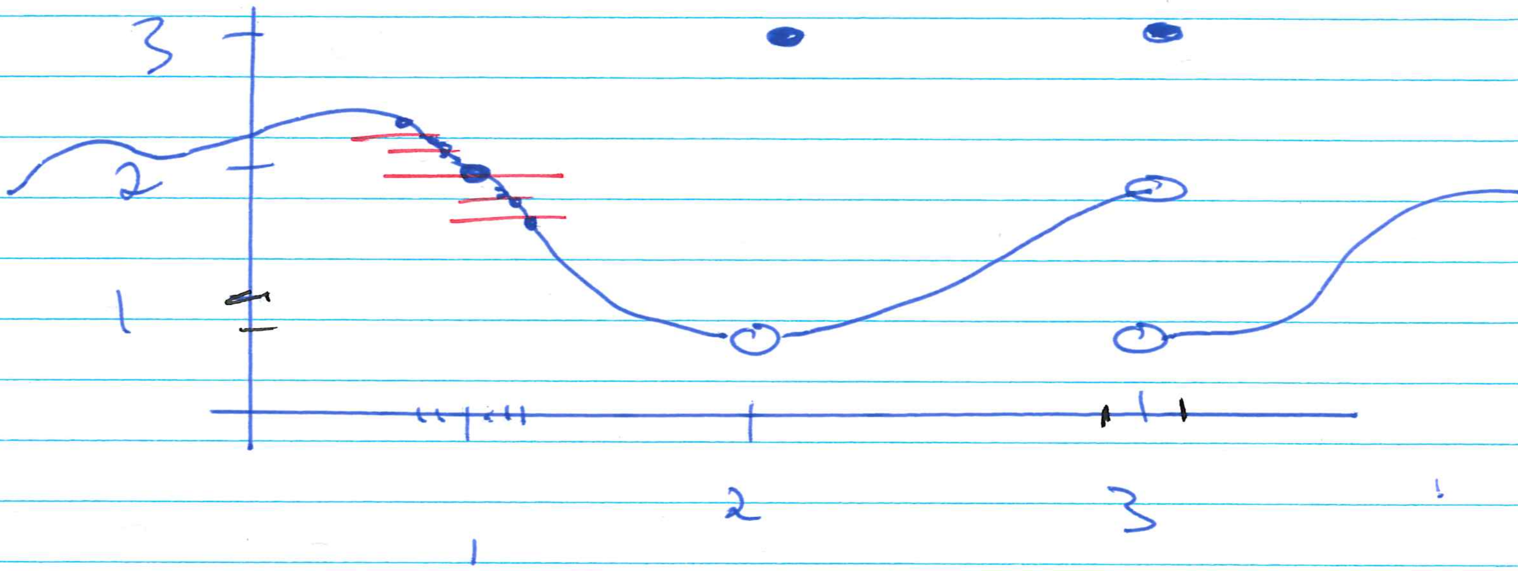
$$\lim_{x \to 1} \left[ f(x) \right] = 2.$$

Note, however, that $f(1) = 3 \neq 2$.

④

The limit cares not for what happens at $x=1$, just near $x=1$.

Example: graph of $f(x)$.



Clicker: What is $\lim_{x\to 1} f(x)$? $= 2$.

A) 1

B) 2

C) 3

D) None of the above.

E) No idea.

Clicker: $\lim_{x\to 2} f(x)$? $= 1$.

A.

5

Clicker Q: $\lim_{x \to 3} f(x)$ ?

D.

This limit does not exist.
there is no one number that $f(x)$ approaches.

$\lim_{x \to 3} f(x)$ does not exist (D.N.E.).

This function does have one sided limits at $x = 3$.

$\lim_{x \to 3^-} f(x) = 2$, $\lim_{x \to 3^+} f(x) = 1$.

one sided limit from the left/below.

One sided limit from the right/above.

Let's find the one sided limits at $x = 2$.

$\lim_{x \to 2^-} f(x) = 1$, $\lim_{x \to 2^+} f(x) = 1$.

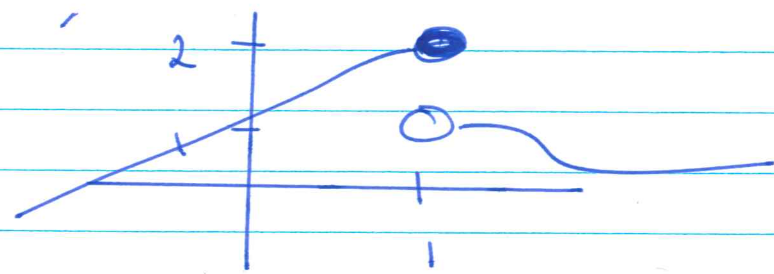
⑥

Note: If the one sided limits exist and are the same number then the full limit exists and is that same number.

ie. $\lim_{x \to 2^-} f(x) = 1$, $\lim_{x \to 2^+} f(x) = 1$.

implies, $\lim_{x \to 2} f(x) = 1$.

If the two one sided limits are different then the full limit D.N.E.

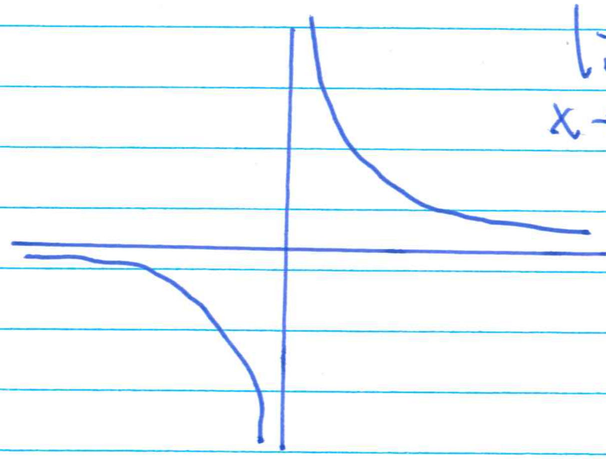



$\lim_{x \to 1^-} f(x) = 2$, $\lim_{x \to 1^+} f(x) = 1$.

So, $\lim_{x \to 1} f(x)$ D.N.E.

7.

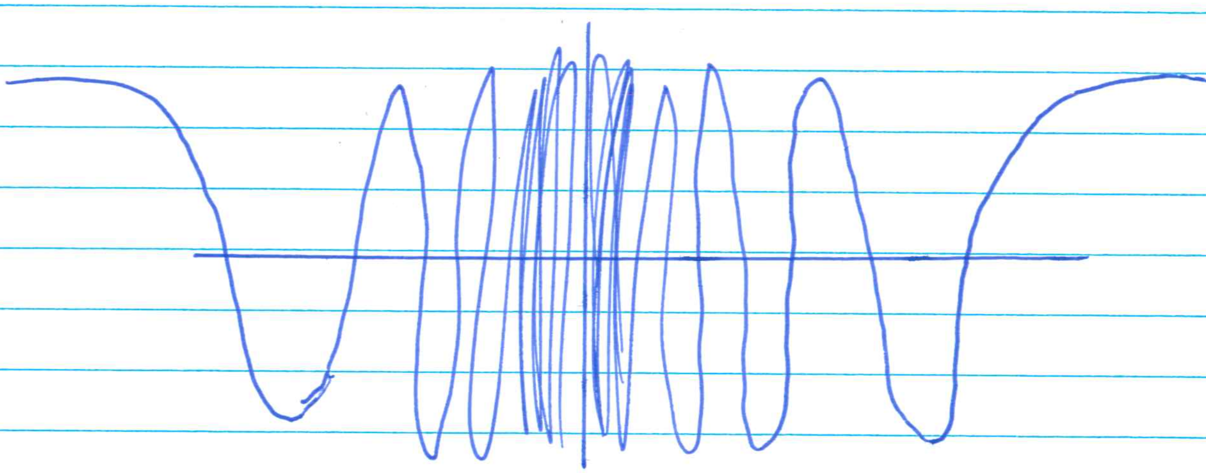
Can anyone think of a function where the one sided limits do not exist?

- 

$\lim_{x \to 0^+} \frac{1}{x}$ D.N.E.

(asymptotes later this week.)

- $\cos\left(\frac{1}{x}\right)$



$\lim_{x \to 0^+} \cos\left(\frac{1}{x}\right)$ does not exist.

there is no number the y value approaches as $x \to 0^+$.

Also $\lim_{x \to 0^-} f(x)$ DNE. $\lim_{x \to 0} \cos\left(\frac{1}{x}\right)$ DNE.