①

Sept 29

- HW 1 is due
  - Solutions later today.
- HW 2 is posted.
- Quiz #1 is Friday
  - functions
  - trig.
- Practice Problems posted.

One more trig. problem.

Example: Find x in $[0, 2\pi)$ satisfying

$$2\sin^2 x + \sin x - 1 = 0.$$

To factor, make substitution.

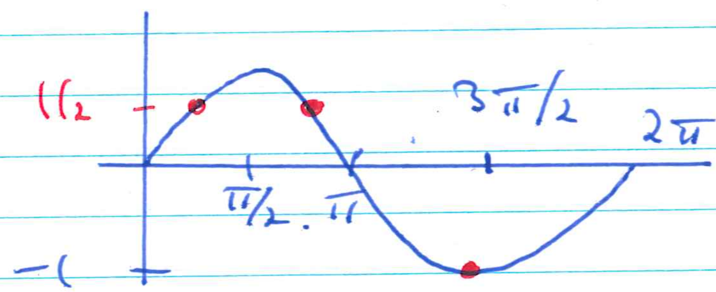
Let $u = \sin x$

So
$$2u^2 + u - 1 = 0.$$
$$2u^2 + 2u - u - 1 = 0$$
$$2u(u+1) - 1(u+1) = 0.$$
$$(u+1)(2u-1) = 0.$$

$$u = -1, \ 1/2.$$

So, $\sin x = -1$, $\sin x = 1/2$.

For $\sin x = -1$.

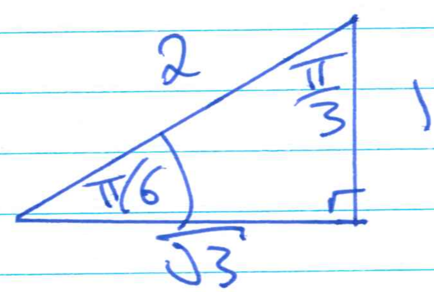


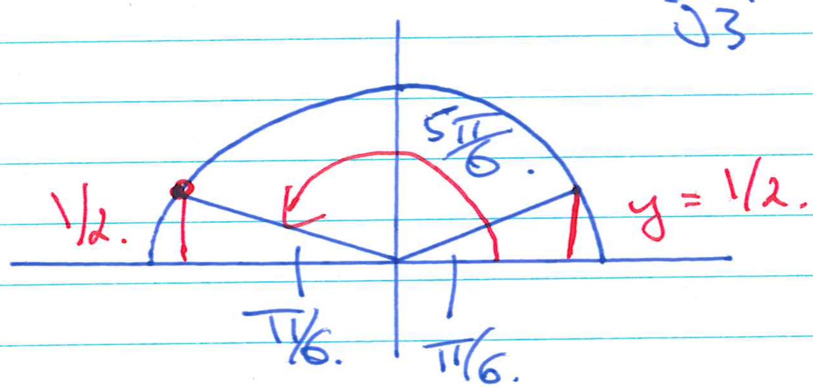
So $x = 3\pi/2$.

"SOH".

For $\sin x = 1/2$.

$x = \pi/6$.





$\left(\pi - \pi/6 = \frac{5\pi}{6}\right)$.

$x = 5\pi/6$.

Collecting everything we see

$x = \pi/6,\ 5\pi/6,\ \frac{3\pi}{2}$.

(B)

Exponents and Exponential Functions

Let's think about $f(x) = 2^x$.

(not a polynomial : x^2, x^3)

If $x = a$ is a natural number $(1, 2, 3, 4, \dots)$.

$$2^a = \underbrace{2 \cdot 2 \cdot \dots \cdot 2}_{a \text{ times}}$$

Observe

$$\boxed{2^a \cdot 2^b = \overbrace{2 \cdot 2 \cdot \dots \cdot 2}^{a} \cdot \overbrace{2 \cdot 2 \cdot \dots \cdot 2}^{b} = 2^{a+b}}$$

and

$$\boxed{\begin{aligned} (2^a)^b &= \underbrace{2^a \cdot 2^a \cdot \dots \cdot 2^a}_{b \text{ times}}. \\ &= \underbrace{\underbrace{2 \cdot 2 \cdot \dots \cdot 2}_{a \text{ times}} \cdot \underbrace{2 \cdot 2 \cdot \dots \cdot 2}_{a \text{ times}} \cdot \dots \cdot \underbrace{2 \cdot \dots \cdot 2}_{a \text{ times}}}_{b \text{ times}}. \\ &= 2^{ab} \end{aligned}}$$

④

What is 2^{-1}?

$$2^{-1} = 1/2.$$

$$2^{-2} = 1/2^2 = 1/4.$$

So, $$\boxed{\frac{2^a}{2^b} = 2^a \cdot 2^{-b} = 2^{a-b}.}$$

What is $2^{1/2}$?

$$2^{1/2} = \sqrt{2}.$$

$$2^{1/3} = \sqrt[3]{2}.$$

$$2^{1/a} = \sqrt[a]{2}.$$

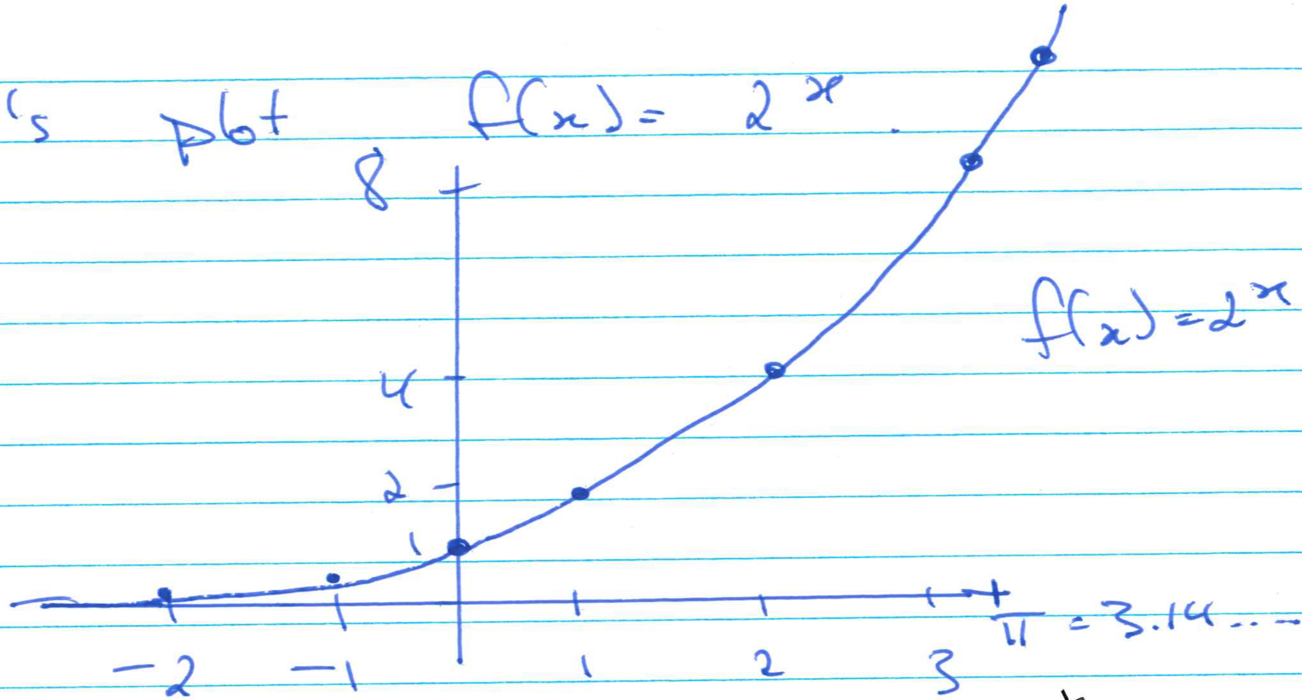
$$\boxed{2^{a/b} = \left(\sqrt[b]{2}\right)^a = \sqrt[b]{2^a}.}$$

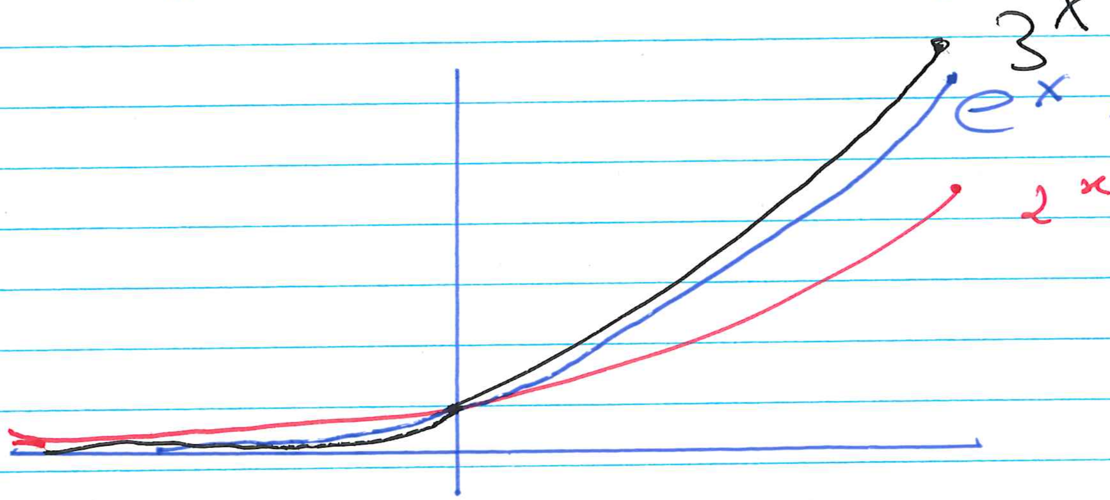
What about 2^0?

$$\boxed{2^0 = 2^1 \cdot 2^{-1} = \frac{2}{2} = 1.}$$

(5)

Let's plot $f(x) = 2^x$.





What is e? $e = 2.71\ldots$

Now $f(x) = e^x$ is an incredibly important function. (more on why later).

Example: Simplify

$$\frac{(\sqrt{e}\cdot e^2)^3}{e}.$$

$$= \frac{(e^{1/2}\cdot e^2)^3}{e}.$$

$$\frac{(e^{2+1/2})^3}{e}$$

$$\| $$

$$\frac{(e^{5/2})^3}{e}.$$

$$\| $$

$$\frac{e^{15/2}}{e}$$

$$\frac{e^{3/2}\cdot e^6}{e}.$$

$$\| $$

$$\frac{e^{15/2}}{e}.$$

$$e^{\frac{15}{2}}e^{-1}$$

$$\| \; e^{13/2}.$$

$$(e^{1/2}\cdot e^2)^3\cdot e^{-1}.$$