1

Oct-5

- Quiz #2 - Friday. - limits.

Examples: $f(x) = \dfrac{-x^2+7}{2x^2+5x}$.

Find all Horizontal Asymptotes

Let us compute • $\lim_{x \to \infty} f(x)$.

• $\lim_{x \to -\infty} f(x)$.

$\lim_{x \to \infty} \dfrac{-x^2+7}{2x^2+5x}$. (divide each term by x^2)

$= \lim_{x \to \infty} \dfrac{-x^2/x^2 + 7/x^2}{2x^2/x^2 + 5x/x^2}$

$= \lim_{x \to \infty} \dfrac{-1 + 7/x^2}{2 + 5/x}$.

both go to zero as $x \to \infty$.

$= ~~\lim_{x \to \infty}~~ \dfrac{-1+0}{2+0} = -1/2$.

$\Rightarrow$ H.A. at $y = -1/2$.

Similarly, $\lim_{x \to -\infty} f(x) = -1/2$.

②

Now, $g(x) = \dfrac{-2}{e^x+3}$.

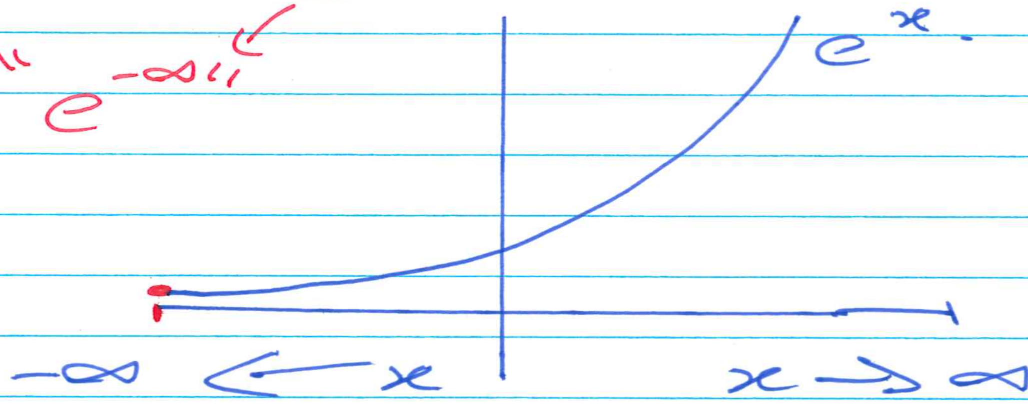
- $\displaystyle\lim_{x\to\infty} \frac{-2}{e^x+3} = 0$.

$\Rightarrow$ H.A. at $y=0$.

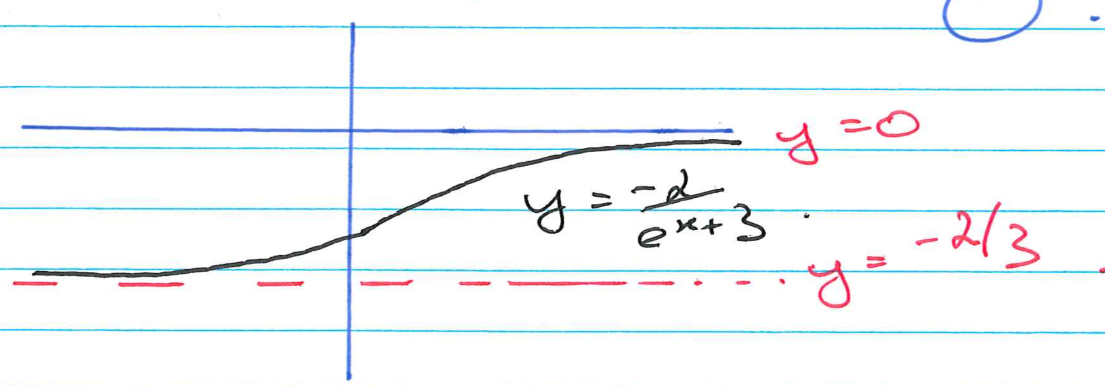
- $\displaystyle\lim_{x\to-\infty} \frac{-2}{e^x+3} = -2/3$. $\Rightarrow$ H.A. at $y = -2/3$.

"$\frac{-2}{0+3}$."

"$e^{-\infty}$"



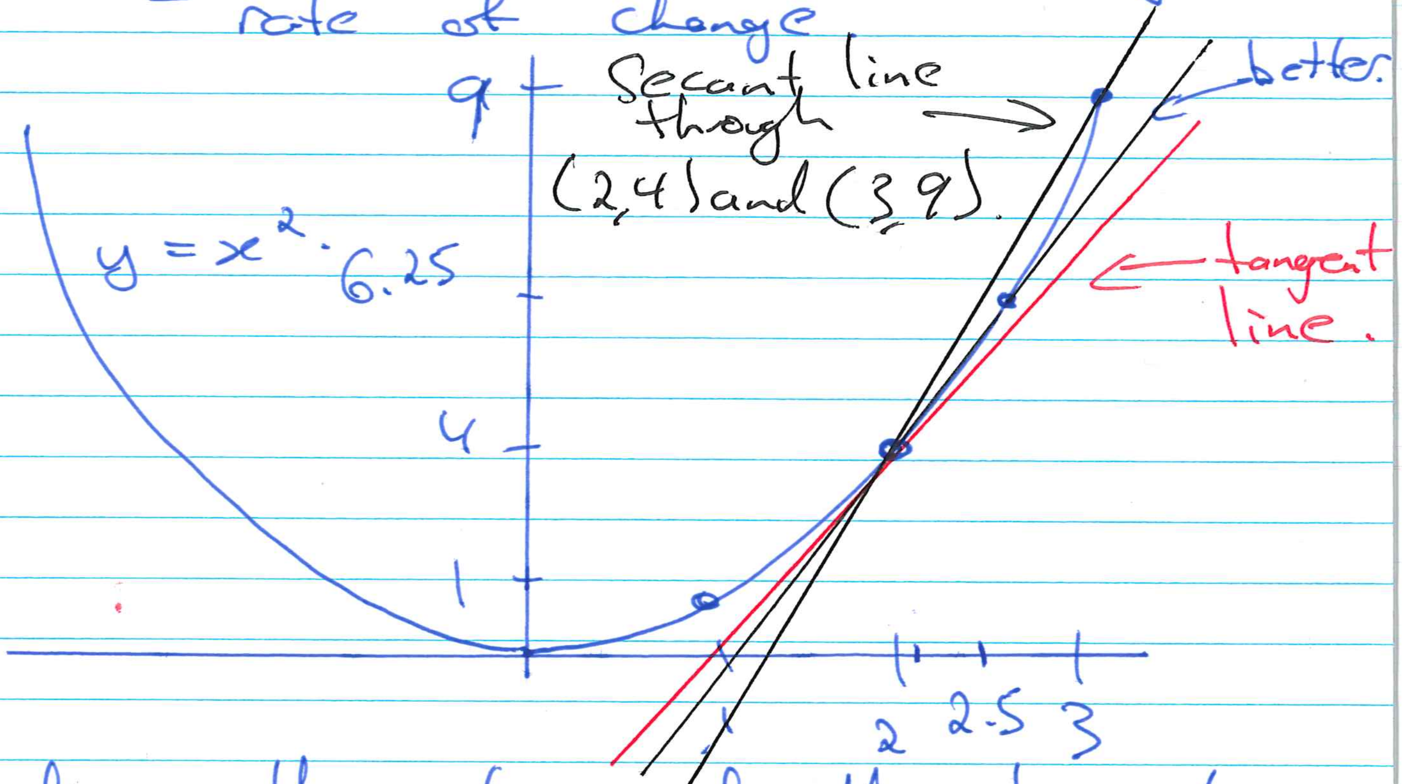
- $\displaystyle\lim_{x\to\infty} e^x = \infty$.
- $\displaystyle\lim_{x\to-\infty} e^x = 0$.



③

Slope of the Tangent Line (§2.0)

We want to find the slope of the tangent line.
- Speed (instantaneous velocity).
- rate of change



Finding the slope of the tangent line is hard since we only have one point.

Easier is finding the slope of the secant line.

4

Find the slope of secant line through $(2,4)$ and $(3,9)$.
(x_1, y_1) (x_2, y_2).

$$m_{sec} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{9-4}{3-2} = \frac{5}{1} = 5.$$

crude approximation of the tangent line.

A better approximation would be to use $(2,4)$ and $(2.5, 6.25)$.

$$m_{sec} = \frac{6.25-4}{2.5-2} = \frac{2.25}{0.5} = 4.5.$$

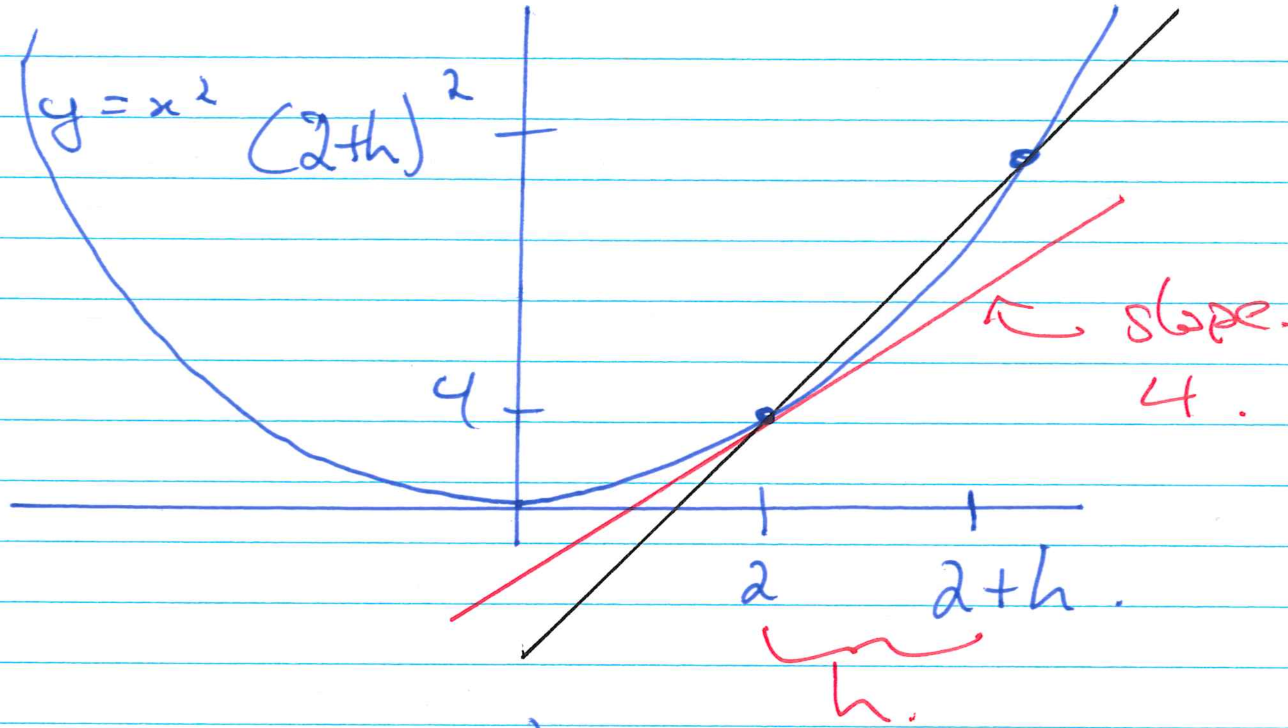
better.

Even better is $(2.1, 4.41)$

$$m_{sec} = \frac{4.41-4}{2.1-2} = \frac{0.41}{0.1} = 4.1.$$

even better.

To find the slope exactly we take the limit.



$$m_{sec} = \frac{(2+h)^2 - 4}{2+h-2}$$

$$= \frac{(2+h)^2 - 4}{h}.$$

As h gets smaller our m_{sec} gets closer to m_{tan}. Consider the limit.

$$m_{tan} = \lim_{h \to 0} m_{sec} = \lim_{h \to 0} \frac{(2+h)^2 - 4}{h}.$$

- Substitution will not work. "$\frac{0}{0}$"

do some algebra.

⑥ $(2+h)^2 = (2+h)(2+h) = 4 + 2h + 2h + h^2$.

$$= \lim_{h\to 0} \frac{\cancel{4} + 4h + h^2 \cancel{-4}}{h}.$$

$$= \lim_{h\to 0} \frac{4h + h^2}{h}.$$

$$= \lim_{h\to 0} \frac{\cancel{h}(4+h)}{\cancel{h}}.$$

$$= \lim_{h\to 0} (4+h).$$

$= 4.$ ← slope of the tangent line at $x = 2$.

Example: Find the slope of the tangent line to x^2 at the point $(3, 9)$.

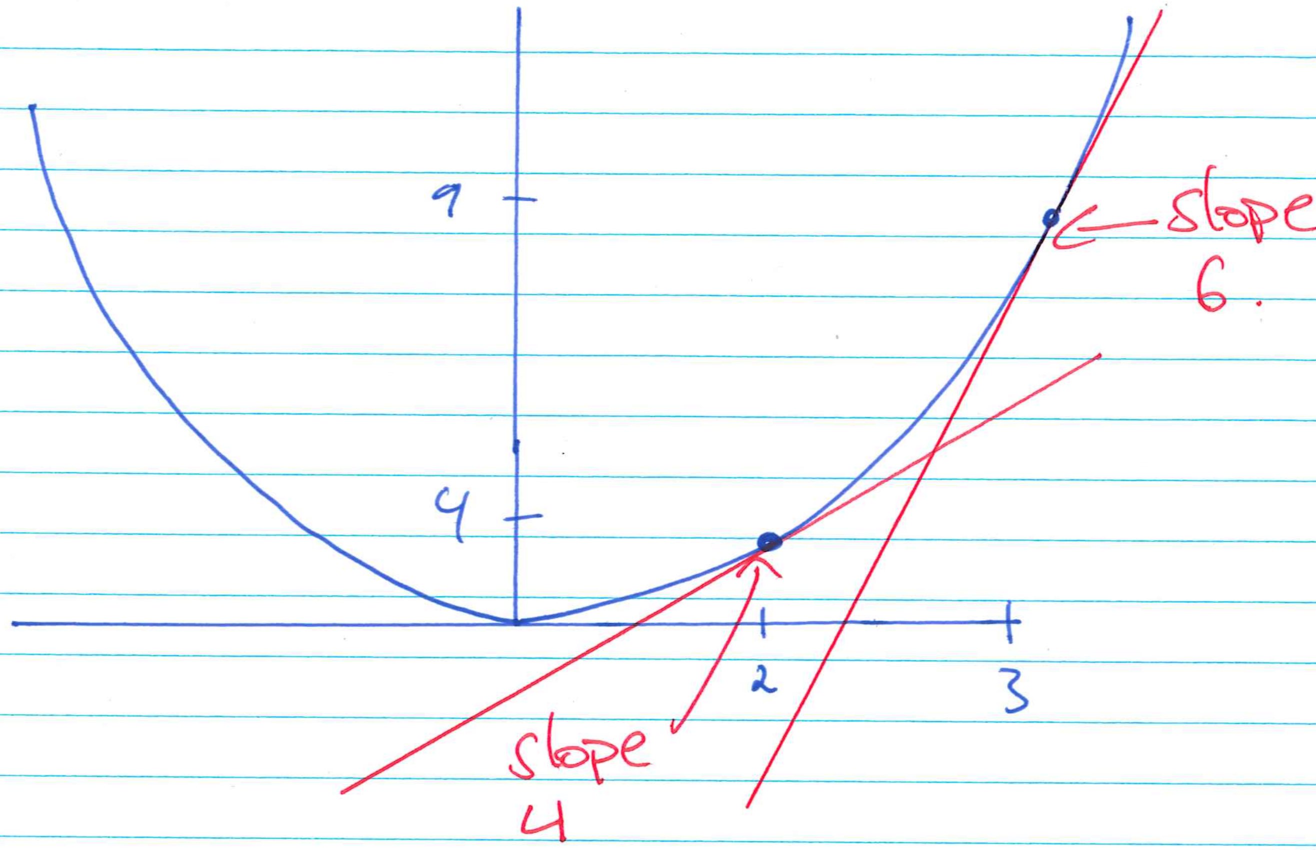
$$m_{tan} = \lim_{h\to 0} \frac{(3+h)^2 - 9}{3+h-3}.$$

$$= \lim_{h\to 0} \frac{(3+h)^2 - 9}{h}.$$

$$= \lim_{h\to 0} \frac{\cancel{9} + 6h + h^2 \cancel{-9}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(6+h)}{\cancel{h}}$$

$$= \lim_{h\to 0} (6+h) = 6 + 0 = 6 //.$$


9
4
2
3
slope 6.
slope 4