

①

Oct. 3.

- Quiz 2 Friday
- limits (last week)
- HW 3 Due today
- HW 4 posted (due next Wed.)
- HW 2 / Quiz 1 returned.

Recall from last class that a function has a vertical asymptote at $x = a$ if:

one of (or both)

- $\lim_{x \rightarrow a^-} f(x) = +\infty$ or $-\infty$
- $\lim_{x \rightarrow a^+} f(x) = +\infty$ or $-\infty$.

Example: Find all vertical asymptotes of

$$f(x) = \frac{x+1}{x^2-x-2}$$

$$= \frac{x+1}{(x+1)(x-2)}$$

$$= \begin{cases} \frac{1}{x-2}, & x \neq -1 \\ \text{undefined}, & x = -1 \end{cases}$$

2
Candidates for vertical asymptotes:

$$x = -1 \quad x = 2$$

Let's take one sided limits

$$\bullet \lim_{x \rightarrow -1^-} \frac{\cancel{(x+1)}}{\cancel{(x+1)}(x-2)} = \lim_{x \rightarrow -1^-} \frac{1}{x-2} = -\frac{1}{3}$$

$$\bullet \lim_{x \rightarrow -1^+} \frac{\cancel{(x+1)}}{\cancel{(x+1)}(x-2)} = \lim_{x \rightarrow -1^+} \frac{1}{x-2} = -\frac{1}{3}$$

\Rightarrow no vertical asymptote at $x = -1$ not $+\infty$ or $-\infty$.

For $x = 2$:

$$\bullet \lim_{x \rightarrow 2^-} \frac{\cancel{(x+1)}}{\cancel{(x+1)}(x-2)} = \lim_{x \rightarrow 2^-} \frac{1}{x-2} = -\infty$$

$$\left(\frac{+}{-} \right)$$

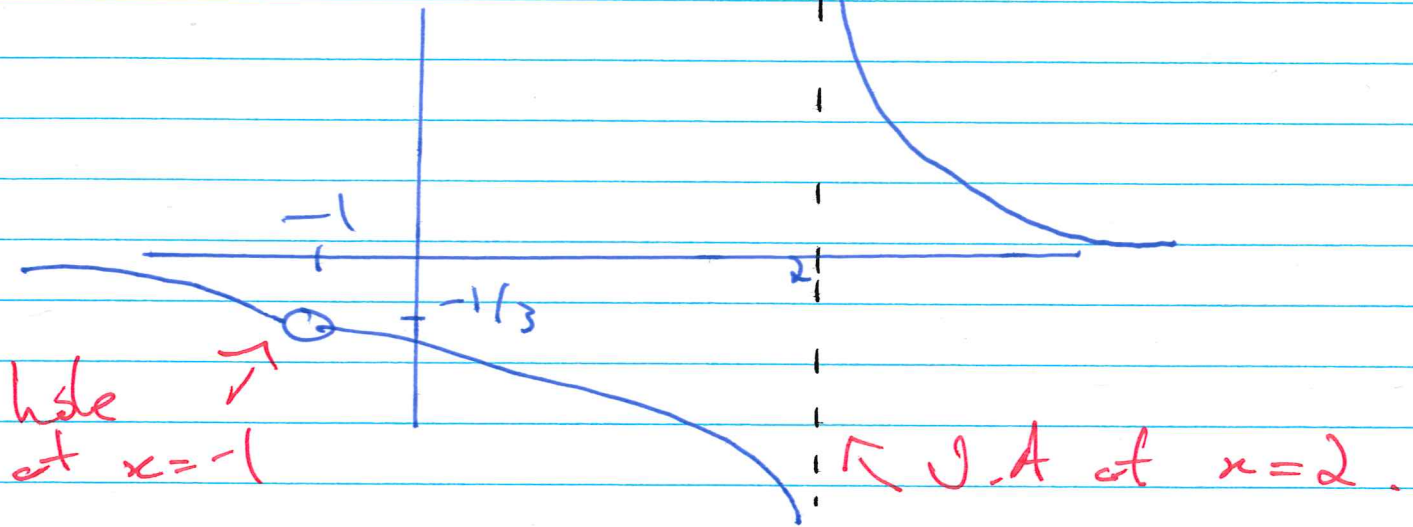
$$\bullet \lim_{x \rightarrow 2^+} \frac{\cancel{(x+1)}}{\cancel{(x+1)}(x-2)} = \lim_{x \rightarrow 2^+} \frac{1}{x-2} = +\infty$$

$$\left(\frac{+}{+} \right)$$

③

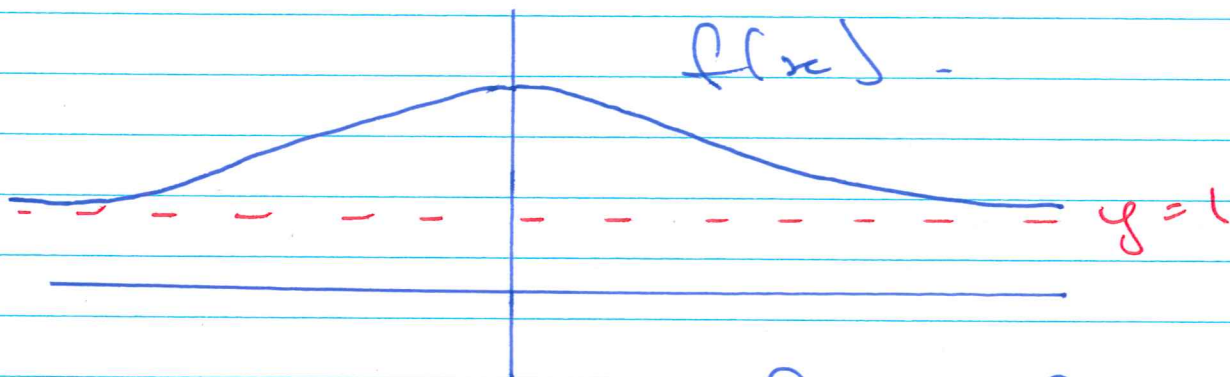
\Rightarrow We have a V.A. at $x=2$.

By the way,



~~graph~~

Horizontal Asymptotes.



What does it mean for a function to have a horizontal asymptote?

In the above example we write,

$$\bullet \lim_{x \rightarrow \infty} f(x) = 1$$

$$\bullet \lim_{x \rightarrow -\infty} f(x) = 1.$$

We say, as x gets large (or largely 'negative') $f(x)$ gets 'close to 1'.

3

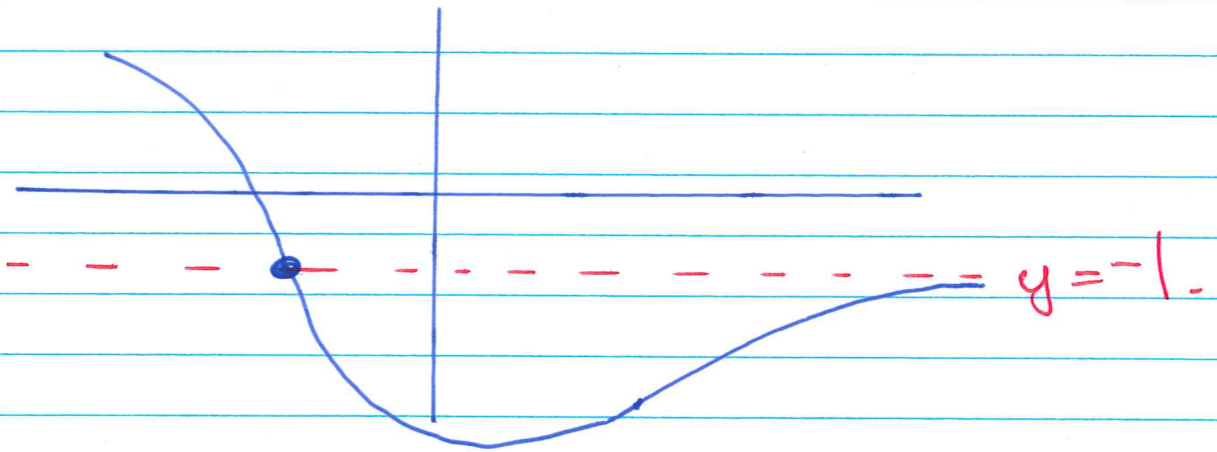
In general,

If $\lim_{x \rightarrow \infty} f(x) = L$ or $\lim_{x \rightarrow -\infty} f(x) = L$

we say $f(x)$ has a horizontal asymptote at $y = L$.

Check Q: Can a function cross its horizontal asymptote?

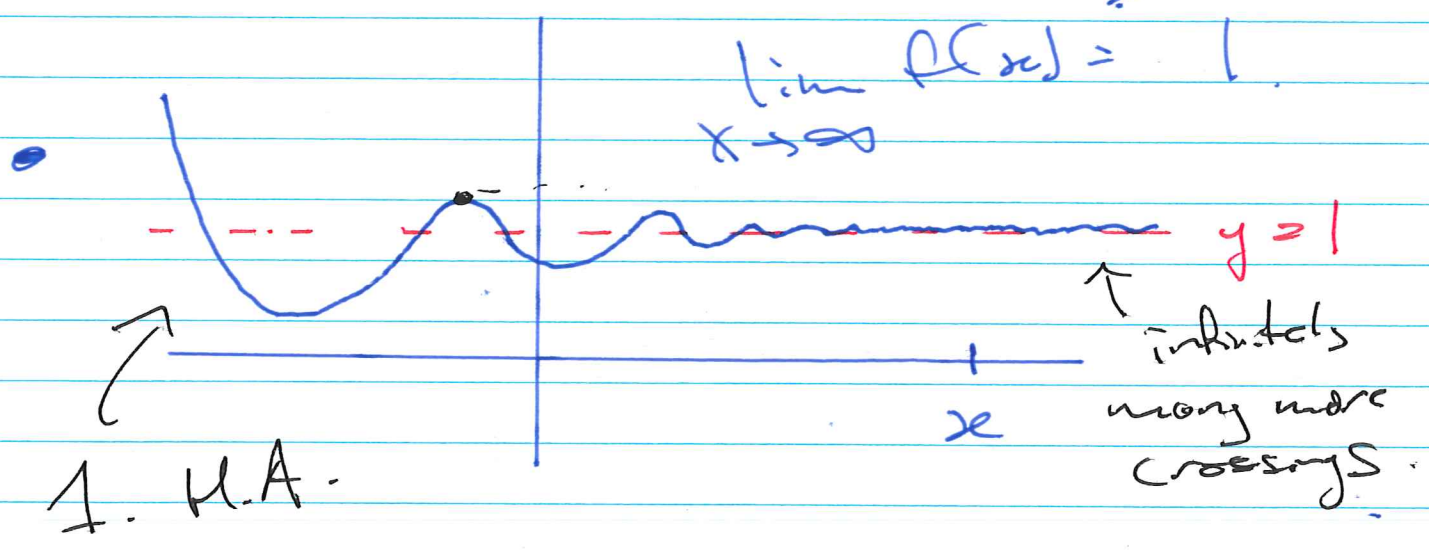
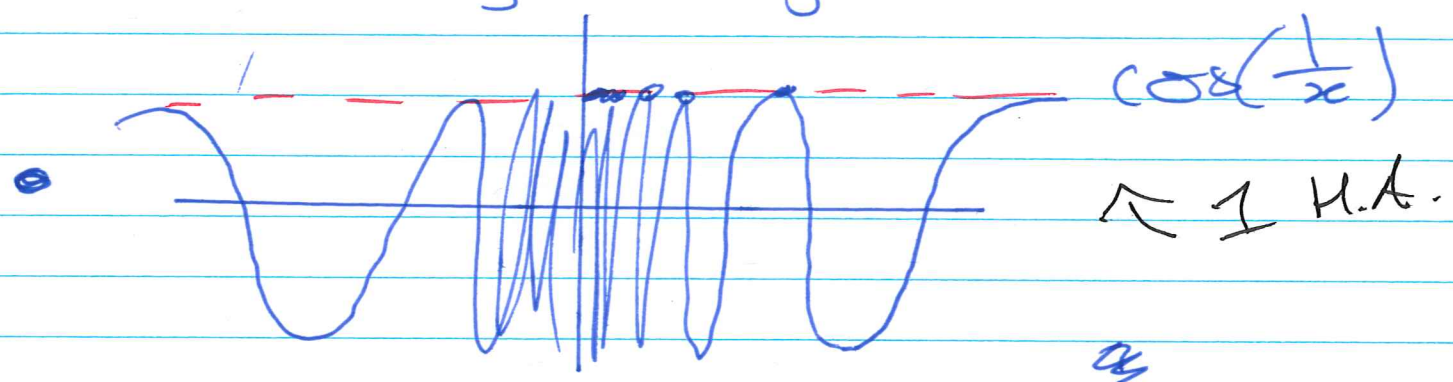
- A) Yes 33%
B) No 67%
C) Don't know.



6

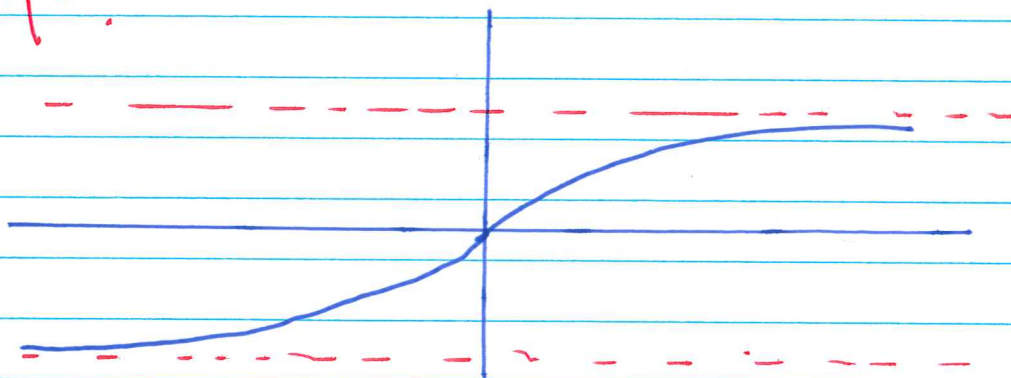
Click Q: How many times can it cross its h.A.

- A) 1
- B) 2
- C) a few times (finitely many).
- D) ~~infinitely many times~~
 → infinitely many times.



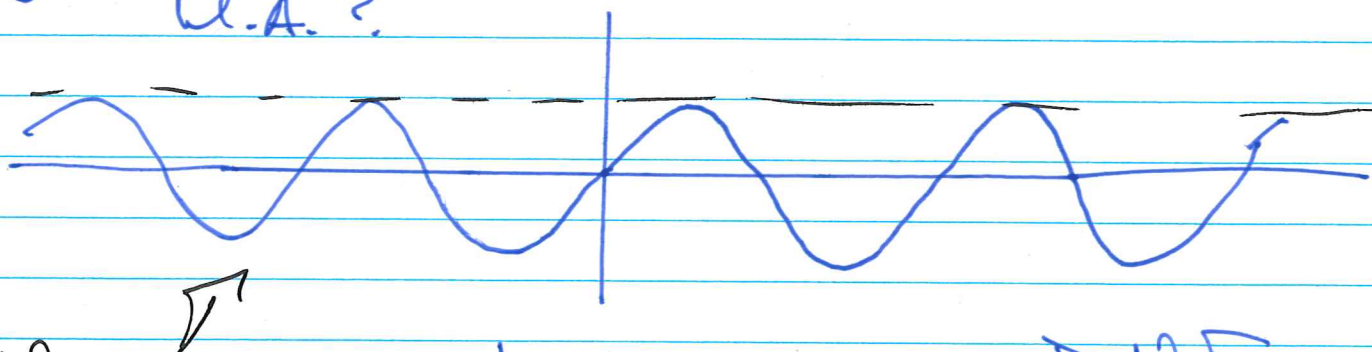
Clicker Q = What is the max number of H.A. a function can have.

- A) 0 D) 3
B) 1 E) Infinitely many
C) 2



2.
H.A.

Does the sine function have a H.A.?



No
H.A.

$\lim_{x \rightarrow \infty} \sin x$ D.N.E.

$\lim_{x \rightarrow -\infty} \sin x$ D.N.E.

8-

To find a function's H.A.
(if any). Compute.

$$\lim_{x \rightarrow \infty} f(x) \quad \text{and} \quad \lim_{x \rightarrow -\infty} f(x)$$

Examples:

$$\bullet f(x) = \frac{-x^2 + 7}{2x^2 + 5x}$$

$$\bullet f(x) = \frac{-2}{e^x + 3}$$