Oct. 19.

- Quiz #3 Friday
  - definition of derivative
  - power/product rule.

- Midterm: Oct. 31

Chain Rule (§ 2.4)

$h(x) = \sin(x^2)$ → composition of two functions.

Let $h(x) = f(g(x))$.

then, $h'(x) = f'(g(x)) \cdot g'(x)$.

"Take the derivative of the outside, leave the inside the same multiply by the derivative of the inside."

For Example: $f(g(x)) = \sin(x^2)$.

$f(x) = \sin x$ ← outside
$g(x) = x^2$. ← inside.

$f'(x) = \cos x$
$g'(x) = 2x$.

$f'(g(x)) = f'(x^2) = \cos(x^2)$.

2

So, $h'(x) = f'(g(x)) \cdot g'(x)$

$= \cos(x^2) \cdot 2x.$

$= 2x\cos(x^2).$

Example: $h(x) = \sqrt{x^2+1}.$

$= (x^2+1)^{1/2}.$

$f(x) = x^{1/2}$ $\qquad$ $g(x) = x^2+1.$

$f'(x) = \frac{1}{2}x^{-1/2}$ $\qquad$ $g'(x) = 2x.$

$h'(x) = f'(g(x)) \cdot g'(x)$

$= \frac{1}{2}(x^2+1)^{-1/2} \cdot 2x.$

Simplification is not necessary.

$= \dfrac{x}{\sqrt{x^2+1}}.$

③

Examples!

1) $\sin(e^x)$

2) e^{2x}

3) $(2x^3+4)^7$

Clickers!

A) 0 done

B) 1

C) 2

D) 3

E) Stuck.

1) $\left[\sin(e^x)\right]'$

$f(x) = \sin x$
$g(x) = e^x$.

$= \cos(e^x) \cdot e^x$

$= e^x \cos(e^x)$.

$f'(x) = 7x^6$.

3) $\left[(2x^3+4)^7\right]'$

$f(x) = x^7$
$g(x) = 2x^3+4$.

$= 7(2x^3+4)^6 (2x^3+4)'$

$= 7(2x^3+4)^6 \cdot 6x^2$.

$= 6 \cdot 7x^2 (2x^3+4)^6$.

2) $h(x) = e^{2x}$.

$e^{2x} = f(g(x))$.

$f(x) = e^{x}$ $\qquad$ $g(x) = 2x$.

$f(g(x)) = f(2x) = e^{2x}$.

$f'(x) = e^{x}$ $\qquad$ $g'(x) = 2$.

$$h'(x) = f'(g(x)) \cdot g'(x)$$
$$= e^{2x} \cdot 2$$
$$= 2e^{2x}.$$

The derivative of $\ln x$.

Notice that $e^{\ln x} = x$.
Take the derivative of both sides.

$$(e^{\ln x})' = (x)'.$$
$$e^{\ln x}(\ln x)' = 1.$$
$$(\ln x)' = \frac{1}{e^{\ln x}} = \frac{1}{x}.$$

5

So, $\boxed{(\ln x)' = \frac{1}{x}}$.

Ex: $\left[\ln(2x-1)\right]'$:

$f(x) = \ln x \qquad g(x) = 2x-1$

$f'(x) = \frac{1}{x} \qquad g'(x) = 2.$

$h'(x) = f'(g(x)) \cdot g'(x)$

$= \frac{1}{2x-1} \cdot 2 = \frac{2}{2x-1}.$

6.

Example: $\left[\ln\left(\sqrt{x-2}\right)\right]'$

$$= \frac{1}{\sqrt{x-2}} \cdot \left(\left(\sqrt{x-2}\right)^{1/2}\right)'$$

$$= \frac{1}{\sqrt{x-2}} \frac{1}{2}(x-2)^{-1/2} \cdot (x-2.)'$$

$$= \frac{1}{\sqrt{x-2}} \frac{1}{2} \frac{1}{\sqrt{x-2}} \cdot 1. = \frac{1}{2(x-2)}$$

Alternatively

$$\left\{\ln\left((x-2)^{1/2}\right)\right\}'$$

$$= \left\{\frac{1}{2}\ln(x-2)\right\}'$$

Simplification can help before you take the derivative.

$$= \frac{1}{2}\frac{1}{x-2} \cdot 1.$$

$$= \frac{1}{2(x-2)}.$$