Oct. 17.

- HW5 is due.
- HW6 will be posted today!
- Quiz #3 Friday
  - definition of derivative
  - power/product rule.

Recall:

$$\frac{d}{dx}(e^x) = e^x.$$

$$\frac{d}{dx}(\sin x) = \cos x.$$

$$\frac{d}{dx}(\cos x) = -\sin x.$$

power rule. → $$\frac{d}{dx}(x^n) = nx^{n-1}.$$

product rule. → $$\frac{d}{dx}(f \cdot g) = f'g + fg'.$$

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Practice: 1) $h(x) = \underbrace{7\sin x}_{f(x)} \underbrace{\cos x}_{g(x).}$

2) $h(x) = xe^x$.

1) Find the derivative.

$f(x) = 7\sin x$ $\qquad g(x) = \cos x$

$f'(x) = 7\cos x$ $\qquad g'(x) = -\sin x$.

$$h' = f'g + fg'$$

$$= 7\cos x \cos x + 7\sin x(-\sin x)$$

Simplification is not necessary

$$= 7\cos^2 x - 7\sin^2 x.$$

2) $h(x) = \underbrace{x}_{f(x)} \underbrace{e^x}_{g(x).}$

$f(x) = 1x$ $\qquad g(x) = e^x$

$f'(x) = 1$ $\qquad g'(x) = e^x$.

$$h'(x) = f'(x)g(x) + f(x)g'(x)$$

$$= e^x + xe^x.$$

Similar to product rule is
Quotient Rule.

We may want the derivative of

$$h(x) = \frac{f(x)}{g(x)}.$$

(For example $h(x) = \dfrac{x-1}{x}$

HW?)

Quotient Rule:

$$h'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

"low d-high minus high d-low
square the bottom and away
we go"

Ex: $h(x) = \dfrac{x-1}{x} = \dfrac{f(x)}{g(x)}$.

$f(x) = x - 1 \qquad g(x) = x$

$f'(x) = 1 \qquad g'(x) = 1$.

$$h'(x) = \frac{f'g - fg'}{g^2}$$

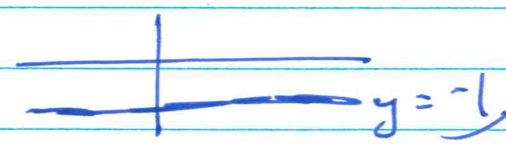
$$= \frac{1 \cdot x - (x-1) \cdot 1}{x^2}.$$

$$= \frac{x - x + 1}{x^2}.$$

as expected.

$$= 1/x^2.$$

(aside: $(x)' = 1 \qquad (-1)' = 0.$



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Example: $h(x) = \dfrac{\sin x}{\cos x} = \tan x$.

Find $h'(x)$. Clicker: A) More time
B) Done
C) Stuck.

$$h'(x) = \frac{(\cos x)(\cos x) - (\sin x)(-\sin x)}{\cos^2 x}.$$

Simplification is not necessary:

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$ ← trig. identity.

$$= \frac{1}{\cos^2 x} \quad (= \sec^2 x).$$ ← derivative of tan x.

(6)

Example: $h(x) = \dfrac{\overbrace{xe^x}^{f(x)}}{\underbrace{2x+1}_{g(x)}}$.

Clickers:
A) Working
B) Done
C) Stuck

$$h'(x) = \frac{(xe^x)'(2x+1) - xe^x(2x+1)'}{(2x+1)^2}.$$

$$= \frac{(1\cdot e^x + xe^x)(2x+1) - xe^x \cdot 2}{(2x+1)^2}.$$

not necessary

$$= \frac{2x^2e^x + \cancel{2xe^x} + xe^x + e^x - \cancel{2xe^x}}{(2x+1)^2}.$$

$$= \frac{2x^2e^x + xe^x + e^x}{(2x+1)^2}.$$

There is one more differential rule we need.
It happens to be the most useful / important.

Chain Rule: (§ 2.4).

We need it to take the derivative of composition of functions.

For example: $h(x) = \sin(x^2)$.

Composition.

outside: $\sin x = f(x)$
inside: $x^2 = g(x)$.

$$h'(x) = f'(g(x)) \cdot g'(x).$$