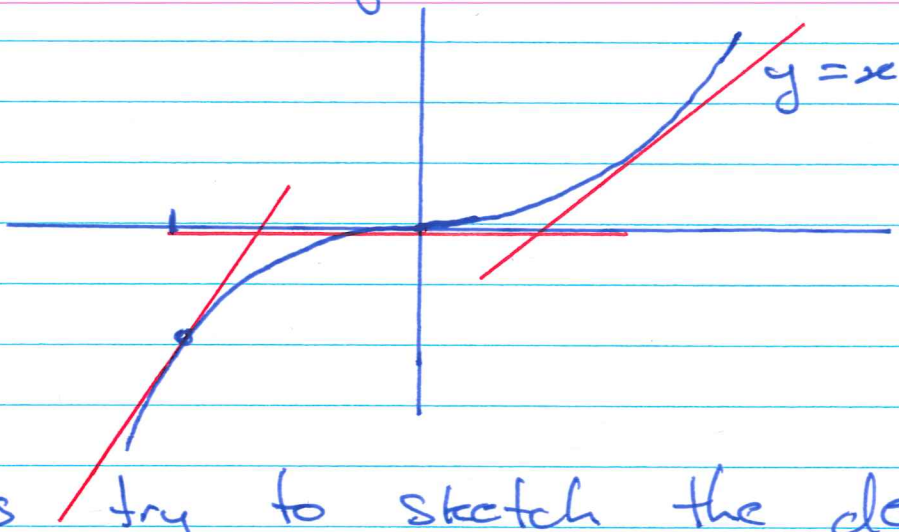
grab a survey → Oct. 12.

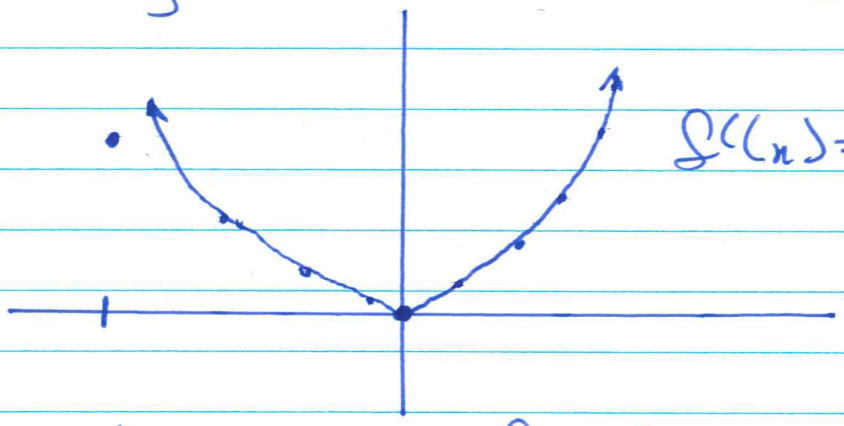
- HW4 Due Today.
- HW5 (short) Due Monday.
- HW3 returned.

- Quiz #3 - Oct. 21
- Midterm - Oct. 31

Consider the graph of $f(x) = x^3$.



Let us try to sketch the derivative $f'(x)$.



The derivative is a function which gives the slope of $f(x)$ at each point.

How to get $f'(x) = 3x^2$.

Last class we applied the definition of the derivative (§2.1)

$$\frac{df}{dx} = f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

$$f(x) = x^3.$$
$$f(x+h) = (x+h)^3.$$

$$\left(\frac{0}{0}\right) \quad f'(x) = \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h} \leftarrow \text{sub. fails.}$$

algebra.

$$= \lim_{h \to 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h}$$

$$= \lim_{h \to 0} \frac{3x^2h + 3xh^2 + h^3}{h}.$$

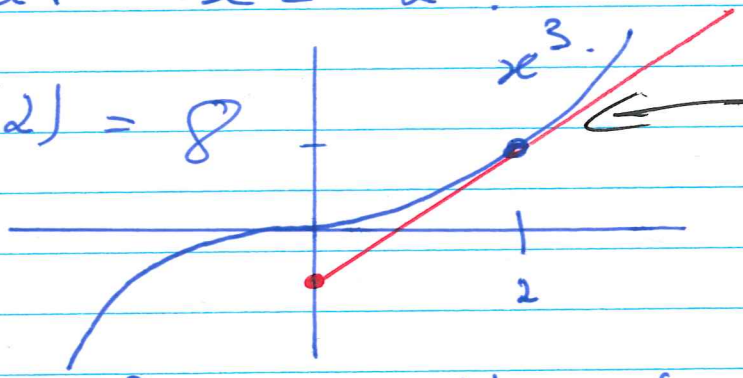
$$= \lim_{h \to 0} \frac{h(3x^2 + 3xh + h^2)}{h}.$$

$$= \lim_{h \to 0} 3x^2 + 3xh + h^2.$$

$$= 3x^2 + 0 + 0 = 3x^2.$$

Example: Find the equation of the line tangent to $f(x) = x^3$ at $x = 2$.

$2^3 = f(2) = 8$



$f'(x) = 3x^2$ gives the slope at x.

At $x = 2$: $f'(2) = 3(2)^2 = 3 \cdot 4 = 12$.

For the equation of the line.
We have slope $m = 12$ at x_1, y_1
and point $(2, 8)$.

$$y - y_1 = m(x - x_1)$$

$$y - 8 = 12(x - 2)$$

Let's collect what we have:

$$(x^2)' = 2x$$

$$(x^3)' = 3x^2.$$

Any guesses for

$$(x^4)' = 4x^3.$$

In general, we have the Power Rule.

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

(n is any real number).

The proof goes like this:

$$\frac{d}{dx}(x^n) = \lim_{h \to 0} \frac{(x+h)^n - x^n}{h}.$$

a bunch of algebra. $\longrightarrow$ $=$ .....

$$= nx^{n-1}.$$

Example: Use power rule to find the derivative of $f(x) = \sqrt{x}$.

(Note: this is done in the Lab using limits)

$$f(x) = \sqrt{x} = x^{1/2}.$$

$$f'(x) = \frac{1}{2} x^{-1/2} = \frac{1}{2\sqrt{x}}.$$

Example: $f(x) = \frac{1}{x}$ (again, done in Lab using limits)

$$f(x) = \frac{1}{x} = x^{-1}$$

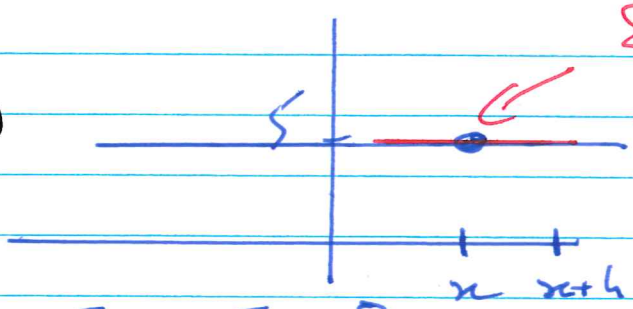
$$f'(x) = -x^{-2} = \frac{-1}{x^2}.$$

ClickerQ: What is the derivative of $f(x) = 5$?

→ A) 0 C) $5x$

B) 5 D) $1/5x$.

(6)

1) 

Slope of tangent is zero.

2) $f(x) = 5 = 5x^0$

power rule.

$f'(x) = 5 \cdot 0\, x^{-1} = 0$.

3) Limit Definition:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{5 - 5}{h}$$

$$= \lim_{h \to 0} \frac{0}{h}$$

$$= 0.$$

Note: the derivative of any constant is zero.

(7).

Find the derivative.

Example: • $f(x) = x^{\pi}$, $f'(x) = \pi x^{\pi-1}$.

• $f(x) = \frac{1}{x^2} + x^{3/2} - x^{2016} + e^{\pi}$.

$= x^{-2} + x^{3/2} - x^{2016} + e^{\pi}$. ($e^{\pi}$: constant / number.)

$f'(x) = -2x^{-3} + \frac{3}{2}x^{1/2} - 2016x^{2015}$

$+ 0$.

Note: We can take the derivative of each term and add them together.