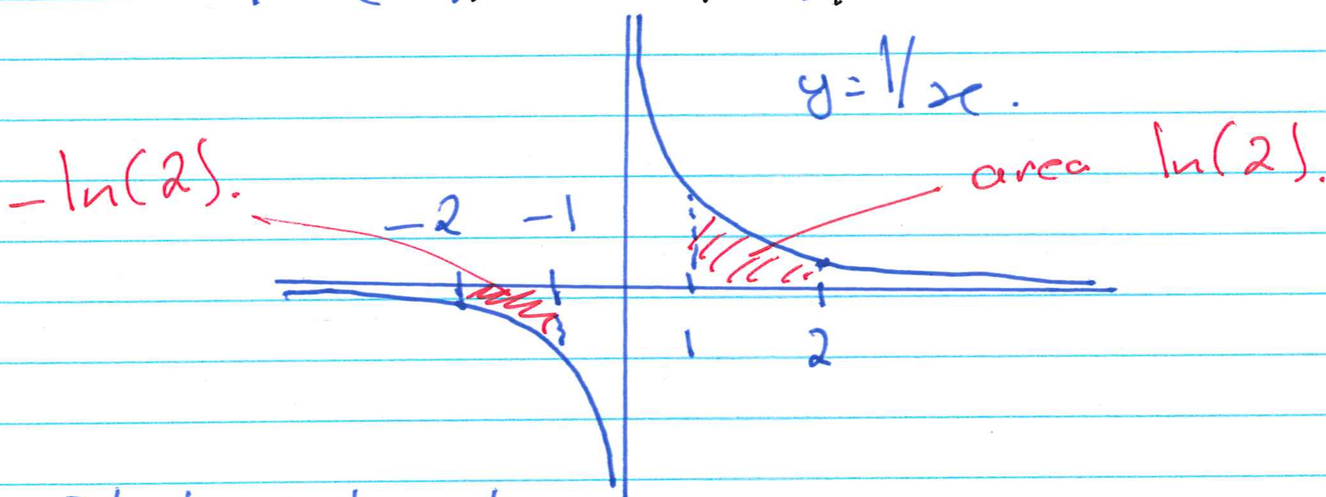
Nov. 9

- Office Hours Wednesday (today) 2–3:30
- Quiz 4 – Monday
  - one problem
  - taken from lab. (15 min)
- HW8 Due Monday.
- No class Friday.

Think about $\int_1^2 \frac{1}{x}\,dx$.

$= \ln|(2)| - \ln|(1)| = \ln(2)$.

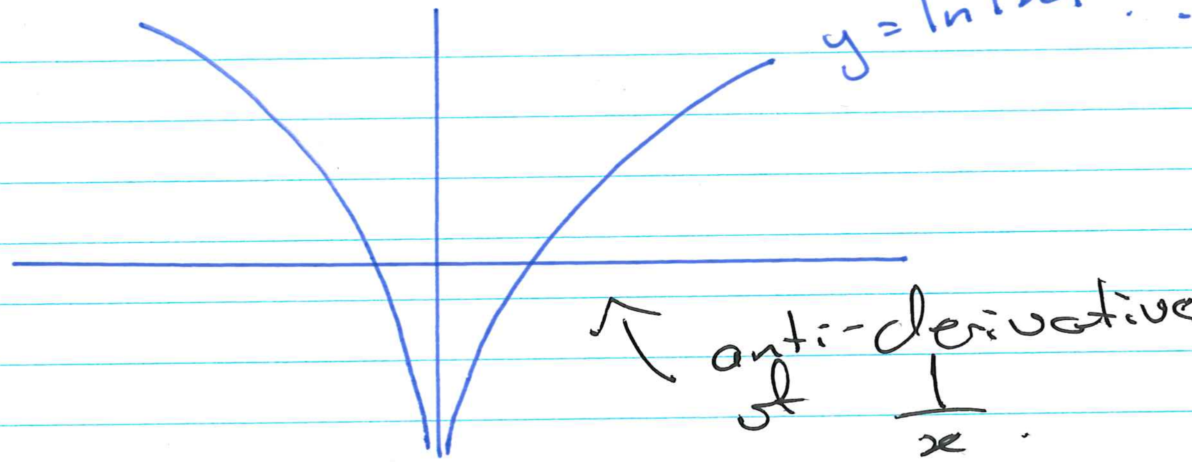


What about

$$\int_{-2}^{-1} \frac{1}{x}\,dx. = \ln|(-1)| - \ln|(-2)|$$

$$= \ln(1) - \ln(2)$$

$$= -\ln(2).$$



So we take $\ln|x|$ as the anti-derivative for $\frac{1}{x}$.

Indefinite Integrals:

ClickerQ: Is $\frac{1}{3}x^3$ the only anti-derivative of x^2?

25%. A) Yes

50%. B) No. ⇐

$$\left[\left(\frac{1}{3}(-x)^3\right)' = \frac{3}{3}(-x)^2 \cdot (-1) = -x^2.\right]$$

What about $\left(\frac{1}{3}x^3 + 1\right)'$

$= x^2$.

(3)

$$\left(\frac{1}{3}x^2 - 2\right)' = x^2.$$ ← derivative will kill constant.

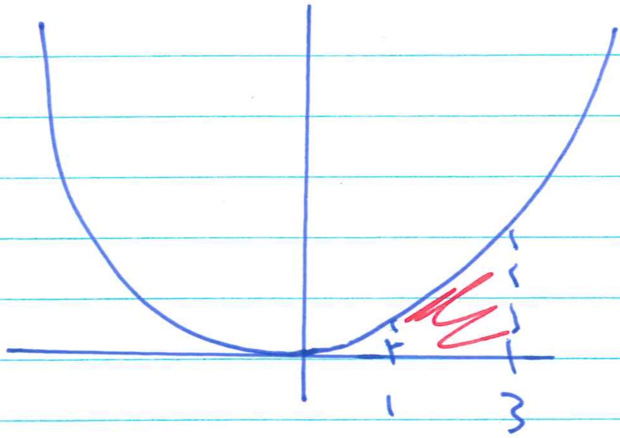
The general anti-derivative of $f(x) = x^2$ is:

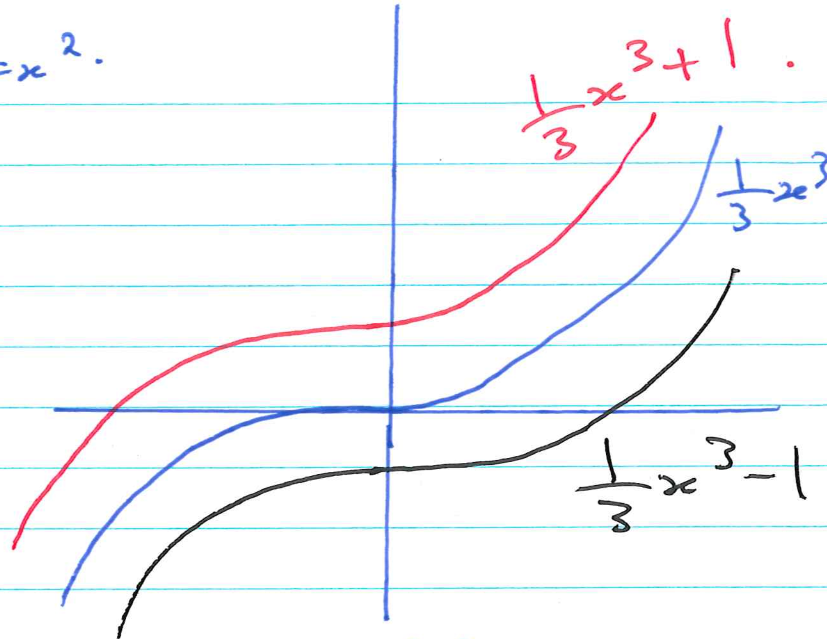
$$F(x) = \frac{1}{3}x^3 + C.$$

where C is a constant.

We also write

- $\int x^2\,dx = \frac{1}{3}x^3 + C.$ ← indefinite integral. ← a function. (a family of functions).
- $\int_1^3 x^2\,dx$ ← a number. ← definite integral.





Does this mess up our definite integrals?

$$\int_1^3 x^2 \, dx = F(b) - F(a)$$

$b = 3$, $a = 1$.

where $F(x)$ is any anti-derivative.

We could use $F(x) = \frac{1}{3}x^3 + 2$.

$F(3) - F(1)$

$= \frac{1}{3}(3)^3 + 2 - \left(\frac{1}{3}(1)^3 + 2\right)$

$= \frac{1}{3}(3)^2 + 2 - \frac{1}{3}(1)^3 - 2$.

Let's record the general anti-derivatives that we know.

- $\int x^n \, dx = \frac{1}{n+1} x^{n+1} + C.$

- $\int \sin x \, dx = -\cos x + C.$

- $\int \cos x \, dx = +\sin x + C.$

- $\int e^x \, dx = e^x + C.$

- $\int \frac{1}{x} \, dx = \ln|x| + C.$

Example: Find the general anti-derivative of $f(x) = \frac{1}{\sqrt{x}} + 5e^x - 2\cos x.$

$$f(x) = \frac{1}{\sqrt{x}} + 5e^{x} - 2\cos x.$$

$$f(x) = x^{-1/2} + 5e^{x} - 2\cos x.$$

$$\int \left(x^{-1/2} + 5e^{x} - 2\cos x \right) dx.$$

$$= 2x^{1/2} + 5e^{x} - 2\sin x + C.$$

Example: Let $f(x) = 2\sqrt{2}\sin x + \sqrt{2}\cos x$.

Find $F(x)$ where $F'(x) = f(x)$.

and $F(\pi/4) = 1$.

$$F(x) = -2\sqrt{2}\cos x + \sqrt{2}\sin x + C.$$

$$1 = F(\pi/4) = -2\sqrt{2}\left(\cos(\pi/4)\right) + \sqrt{2}\sin(\pi/4) + C.$$

$$= -2\sqrt{2}\left(\frac{1}{\sqrt{2}}\right) + \sqrt{2}\left(\frac{1}{\sqrt{2}}\right) + C.$$

$$= -2 + 1 + C$$

$$= -1 + C.$$

$$1 = -1 + C.$$

$$C = 2.$$

So,

$$F(x) = -2\sqrt{2}\cos x + \sqrt{2}\sin x + 2.$$

Specific anti-derivative which satisfies our property $F(\pi/4) = 1$.