(1)

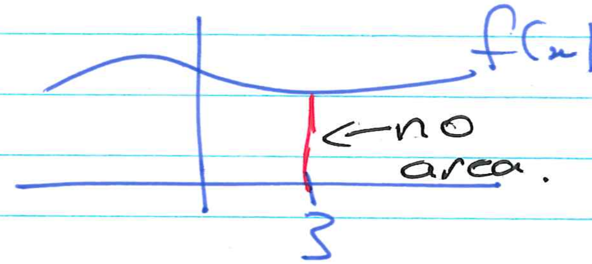
- MAPS (survey) (Dec. 4).
- Final! Dec. 12 at 3:30 in CHBE 101.
- term grades posted Friday.
- C.O. have been tweaked

## Definite Integrals:

anti-derivative

- $\int_a^b f(x)\,dx = F(b) - F(a)$.

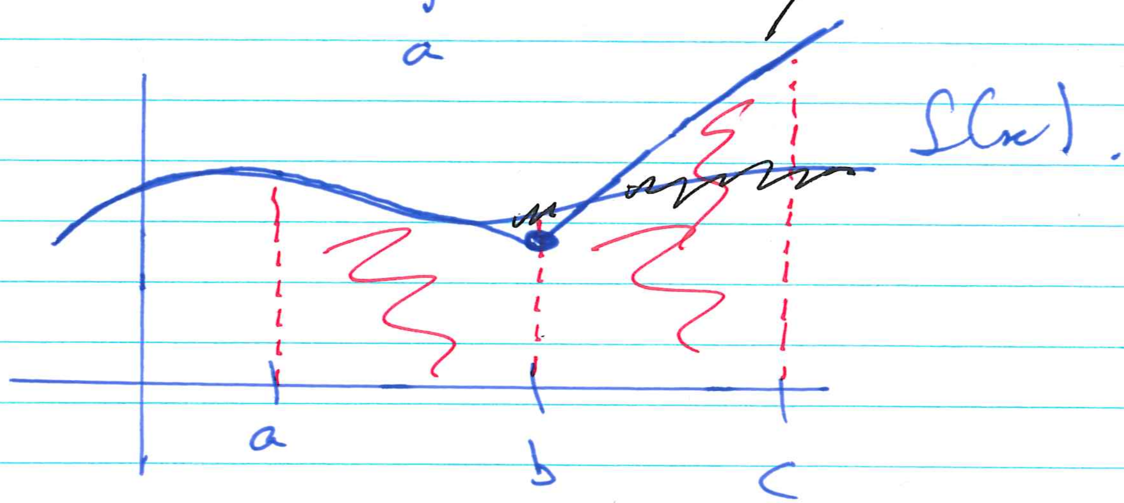
- $\int_3^3 f(x)\,dx = 0$




- $\int_a^b f(x)\,dx + \int_b^c f(x)\,dx$.

$= \int_a^c f(x)\,dx$.

piecewise function.



What about,

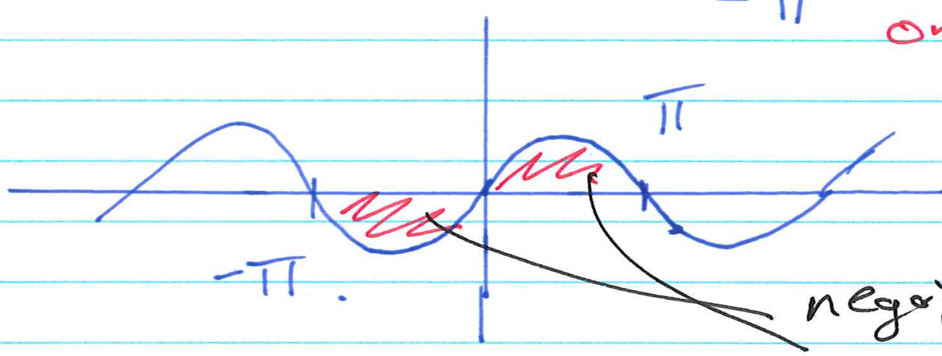
$$\int_a^b f(x)\,dx + \int_b^a f(x)\,dx = \int_a^a f(x)\,dx$$

$$= 0.$$

$$\Rightarrow \underbrace{\int_a^b f(x)\,dx}_{F(b)-F(a)} = \underbrace{-\int_b^a f(x)\,dx}_{-(F(a)-F(b))}.$$

• What about: $\int_{-\pi}^{\pi} \sin x\,dx$.

↑ odd function on symmetric limits.

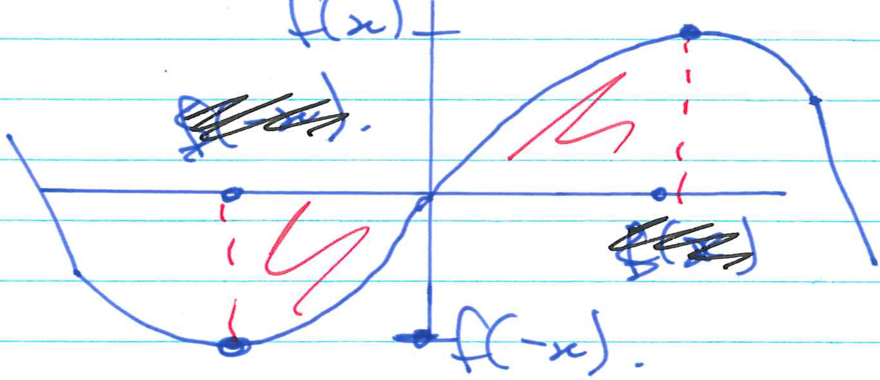



negative and positive area cancel out.

3

## ODD:

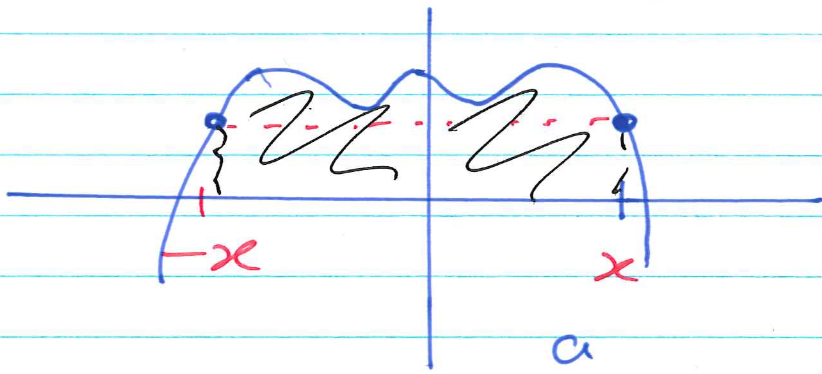
$$f(-x) = -f(x).$$




$$\int_{-a}^{a} f(x)\,dx = 0.$$

## EVEN:

$$f(-x) = f(x).$$




$$\int_{-a}^{a} f(x). = 2\int_{0}^{a} f(x)\,dx.$$

④

EVEN: $f(x) = x^2$.

$$f(-x) = (-x)^2 = x^2 = f(x).$$

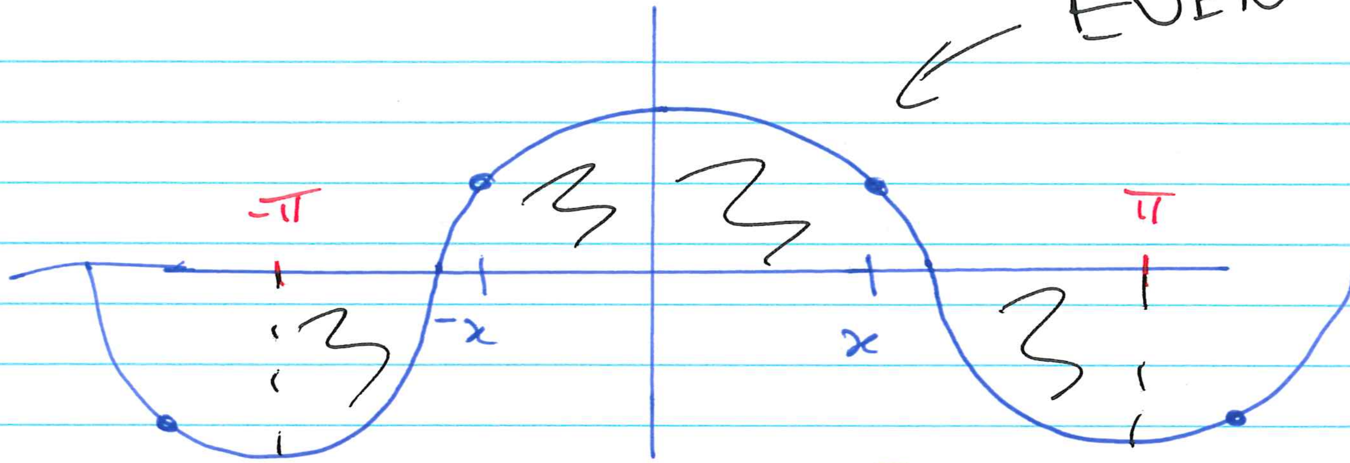
ODD: $f(x) = x$.

$$f(-x) = (-x) = -f(x).$$

HW: $f(x) = x\cos(x^2)$.

$$f(-x) = (-x)\cos((-x)^2).$$
$$= -x\cos(x^2)$$
$$= -f(x).$$

EVEN.




$$\int_{-\pi}^{\pi} f(x)\,dx = 2\int_{0}^{\pi} f(x)\,dx. = 0.$$

Related Rates: Have a rate
want a rate.

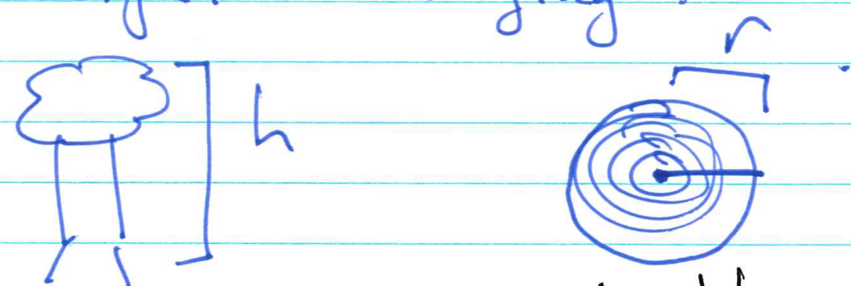
Example: The radius $r$ of a tree grows logarithmically in relation to its height $h$ and can be estimated by the formula

$$r = a \ln(1 + bh).$$

where $a$ and $b$ are positive constants.

If the radius is growing at a rate of 4 cm/year, when the tree is 500 cm tall how fast is the height changing?

1. Picture.




2. Given/Required Rates.

relate these rates

Have: $\frac{dr}{dt} = 4$ cm/year. Want: $\frac{dh}{dt}$.

7

3. Equation that relates $r$ and $h$.

$$r = a \ln(1 + bh)$$

$r = r(t)$ ← functions of time → $h = h(t)$

4. Chain Rule. Take $\frac{d}{dt}$ of both sides.

$$\frac{dr}{dt} = a \frac{d}{dt}\left[\ln(1+bh)\right]$$

$$= a \frac{1}{1+bh} \cdot \frac{d}{dt}(1+bh)$$

$$= a \frac{1}{1+bh} b \frac{dh}{dt}$$

5. Solve / Substitute.

want

$$4 \rightarrow \frac{dr}{dt} = \frac{ab}{1+bh} \frac{dh}{dt}$$

500.

$$\frac{ab}{1+bh}\cdot\frac{dh}{dt} = \frac{dr}{dt}.$$

$$\frac{dh}{dt} = \frac{1+bh}{ab}\,\frac{dr}{dt}.$$

$$= \frac{1+b(500)}{ab}\cdot 4.$$

Come from context.

//

An Integration Word Problem.

A strange alien tree is currently 1 m tall. The rate at which it grows is given by,

growth rate of height. $\rightarrow r(t) = \dfrac{4t}{t^2+1}$ in m/week

How tall is the tree after 10 weeks.

Have the equation of the rate of change. Want info about the actual quantity (height). Integrate!

⑨

$$h(t) = \int r(t)\,dt$$

$$= \int \frac{\overbrace{4t}^{\frac{1}{2}du.}}{\underbrace{t^2+1}_{u.}}\,dt.$$

Let $u = t^2 + 1$

$$\frac{du}{dt} = 2t.$$

$$\frac{1}{2}du = t\,dt.$$

$$h(t) = 4\int \frac{1}{u}\,\frac{1}{2}\,du.$$

$$= 2\int \frac{1}{u}\,du.$$

$$= 2\ln|u| + C.$$

$$h(t) = 2\ln|t^2+1| + C.$$ find $C$.

use: $h(0) = 1.$ (at $t=0$).

(10).

$1 = h(0) = 2\ln|0^2+1| + C$

$= 2\ln(1) + C$

$= 0 + C$

$\Rightarrow C = 1$

$h(t) = 2\ln|t^2+1| + \underline{1}$

Want: $h(10)$

$h(10) = 2\ln|10^2+1| + 1$

$\simeq 10.2\text{ m}$