- Quiz #5 Friday
  - integrals
  - Substitution
- HW9 Due
- HW10 due Mon. Nov. 28
  - last HW for marks.

Integration By Parts (§8.3)

Let's do a few, more complicated, examples.

Example: $\int \underbrace{\ln x}_{u} \cdot \underbrace{1\,dx}_{dv}$.

→ we can integrate the function 1.

$\ln x$ becomes nicer when we differentiate

$u = \ln x$. $\quad dv = 1\,dx$.

$\frac{du}{dx} = \frac{1}{x}$ $\quad \int 1\,dv = \int 1\,dx$.

$du = \frac{1}{x}\,dx$ $\quad v = x$.

$$\int u\,dv = uv - \int v\,du.$$

(2)

$$\int \ln x \, dx = x\ln x - \int x \cdot \frac{1}{x} \, dx.$$

$$= x\ln x - \int 1 \, dx$$

$$= x\ln x - x + C.$$

Check!

$$\left(x\ln x - x + C \right)' = \ln x + x\frac{1}{x} - 1 + 0$$

$$= \ln x + 1 - 1$$

$$= \ln x. \quad //$$

Example: $\int \overbrace{e^x}^{u} \cdot \overbrace{\sin x \, dx}^{dv}$.

Let $u = e^x$

$\frac{du}{dx} = e^x$

$du = e^x dx$.

$dv = \sin x \, dx$.

$\int dv = \int \sin x \, dx$.

$v = -\cos x$.

(37)

$$\int u\,dv = uv - \int v\,du.$$

$$\int e^x \sin x\,dx = -e^x \cos x - \int (-\cos x) e^x\,dx$$

$$= -e^x \cos x + \int \underbrace{e^x}_{u} \underbrace{\cos x\,dx}_{dv}.$$

Did that help? Nicer? Not really.
Try Again? Yes.

Let $u = e^x$ $\quad dv = \cos x\,dx$

$du = e^x dx$ $\quad v = \int \cos x\,dx$

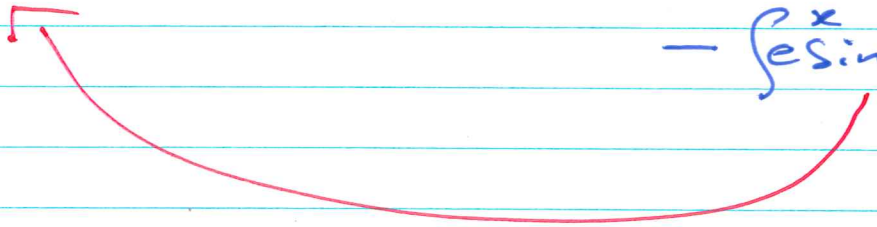
$v = \sin x$.

$$\int e^x \sin x\,dx = -e^x \cos x + \int e^x \cos x\,dx$$

$$= -e^x \cos x + e^x \sin x - \int \sin x\, e^x\,dx.$$

We have,

$$\int e^x \sin x\,dx = -e^x \cos x + e^x \sin x - \int e^x \sin x\,dx.$$



(11)

$$\int e^x \sin x\,dx + \int e^x \sin x\,dx = -e^x \cos x + e^x \sin x.$$

$$2\int e^x \sin x\,dx = -e^x \cos x + e^x \sin x.$$

$$\int e^x \sin x\,dx = \frac{-e^x \cos x + e^x \sin x}{2} + C.$$

Example: We can combine rules.

$$\int x^3 e^{x^2}\,dx.$$

Substitution: $u = x^2 \rightarrow \frac{du}{dx} = 2x \rightarrow \frac{1}{2}du = x\,dx.$

$$\int \underbrace{x^2}_{u}\ \underbrace{e^{x^2}}_{e^u}\ \underbrace{x\,dx}_{1/2\,du}.$$

$$= \frac{1}{2}\int \underbrace{u}_{w}\underbrace{e^u\,du}_{dv}.$$ — IBP.

$$\int w\,dv = wv - \int v\,dw.$$

$$\left(\int u\,dv = uv - \int v\,du\right).$$

(8)

$w = u$ $\qquad dv = e^u du.$

$\frac{dw}{du} = 1.$ $\qquad \int dv = \int e^u du$

$dw = du$ $\qquad v = e^u.$

$$\frac{1}{2}\int ue^u du = \frac{1}{2}\left(ue^u - \int e^u du\right).$$

$$= \frac{1}{2}\left(ue^u - e^u + C\right).$$

Sub: $u = x^2.$

$$= \frac{1}{2}\left(x^2 e^{x^2} - e^{x^2} + C\right).$$