(1)

Substitution Method (for integrals).
(§4.6)

Last Class: $\int x \sin(\overset{u}{\overbrace{x^2}})\, dx$.

x is kind of the derivative of x^2.

If you notice one part of your function as the derivative of another part then a good guess for u is the other part.

Let $u = x^2$. $\frac{du}{dx} = 2x$.

$du = 2x\, dx$.

$\frac{1}{2} du = x\, dx$.

$$\int \sin(\overset{u}{\overbrace{x^2}})\, \overset{1/2\,du}{\overbrace{x\, dx}}.$$

$$= \int \sin(u) \frac{1}{2} du = \frac{1}{2} \int \sin(u)\, du.$$

$$= -\frac{1}{2}\cos(u) + C.$$

$$= -\frac{1}{2}\cos(x^2) + C.$$

Example: $\int \frac{x}{\sqrt{2x^2+3}}\,dx.$

Clicker Q: Let $u = ?$

A) $u = x^2$
→ B) $u = 2x^2 + 3$
C) $u = \sqrt{2x^2+3}$
D) No idea/none of the above.

Let $u = 2x^2 + 3.$

$$\frac{du}{dx} = 4x.$$

$$\frac{1}{4}du = x\,dx.$$

$$\int \frac{1}{\underbrace{\sqrt{2x^2+3}}_{u}} \underbrace{x\,dx}_{\frac{1}{4}du} = \frac{1}{4}\int \frac{1}{\sqrt{u}}\,du.$$

$$= \frac{1}{4} \int u^{-1/2} \, du.$$

$$= \frac{1}{4} \cdot 2 u^{1/2} + C.$$

$$\left(\begin{array}{l} \int u^{-1/2} \, du = \frac{1}{1/2} u^{1/2}. \\ \int u^{n} \, du = \frac{1}{n+1} u^{n+1} \end{array} \right).$$

↑ inverse power rule.

$$= \frac{1}{4} 2 (2x^2+3)^{1/2} + C.$$

$$= \frac{1}{2} \sqrt{2x^2+3} + C.$$

Definite Integrals with Substitution.

$$\int_{\pi/2}^{\pi} \sin^2 x \cos x \, dx .$$

There are two ways we can go.

One: • find indefinite integral
• then do definite integral.

$\int \sin^2 x \cos x \, dx$.

$\begin{cases} u = \sin^2 x \\ u = \sin x \\ u = \cos x \ldots \end{cases}$

$u = \sin^2 x$

• $\frac{du}{dx} = 2 \sin x \cos x$ . . . . . .

$u = \cos x$

• $\frac{du}{dx} = -\sin x$. . . . . . .

• $u = \sin x$. $\frac{du}{dx} = \cos x$.

(5)

$u = \sin x$
$du = \cos x \, dx.$

$$\int \underbrace{\sin^2 x}_{u^2.} \underbrace{\cos x \, dx}_{du}.$$

$$= \int u^2 \, du.$$

$$= \frac{1}{3} u^3 + C.$$

$$= \frac{1}{3} \sin^3 x + C. = F(x).$$

$$\int_{\pi/2}^{\pi} \sin^2 x \cos x \, dx$$

$$= F(\pi) - F(\pi/2)$$

$$= \frac{1}{3} \{\sin(\pi)\}^3 - \frac{1}{3} \{\sin(\tfrac{\pi}{2})\}^3.$$

$\sin(\pi) \to 0$, $\sin(\tfrac{\pi}{2}) = 1.$

~~$+C$~~ ~~$-C$~~.

$$= \frac{1}{3} 0^3 - \frac{1}{3} \cdot 1^3$$

$$= -1/3$$

The other way. Change the limits of integration.

$$\int_{x=\pi/2}^{x=\pi} \underbrace{\sin^2 x}_{u^2} \underbrace{\cos x \, dx}_{du}.$$

let $u = \sin x$
$du = \cos x \, dx.$

when $x = \pi/2$, $u = 1$. $u = \sin(\pi/2) = 1$

when $x = \pi$, $u = 0$. $u = \sin(\pi) = 0.$

$$\int_{u=1}^{u=0} u^2 \, du. = \frac{1}{3}u^3 \Big|_{u=1}^{u=0}.$$

$$= \frac{1}{3}0^3 - \frac{1}{3}1^3$$

$$= -1/3.$$

Example:

$$\int_{1}^{2} \frac{\ln x}{x} \, dx .$$

Next Class.