(1) March 19.

Go over Midterm Session: Wed. 25^{th}. 5-7pm
in LSK 201.

Your midterm mark will be taken
out of 38.

Little bit about Q3.

Show $\frac{d}{dx}(\text{arccot}\, x) = \frac{-1}{1+x^2}$.

$\frac{d}{dx}(\arcsin x)$ also $\frac{d}{dx}(\arctan x)$.

$y = \text{arccot}\, x$

3.b) $\cot y = x$. take $\frac{d}{dx}$:

$\frac{d}{dx}\cot y = \frac{d}{dx}(x)$

$\frac{-1}{\sin^2 y} \cdot \frac{dy}{dx} = 1 \Rightarrow \frac{dy}{dx} = \frac{-1}{\csc^2 y}$.

$-\csc^2 y \frac{dy}{dx} = 1$

$\frac{\sin^2 y + \cos^2 y}{\sin^2 y} = \frac{1}{\sin^2 y}$.

$1 + \cot^2 y = \csc^2 y$.

②

$$\frac{dy}{dx} = \frac{-1}{1+\cot^2 y} = \frac{-1}{1+x^2}.$$

§4.3 Derivatives and Graphs

Last Class: First and Second Derivative tests

Suppose $f'(c) = 0$, c - critical point.

First D.T. If f goes from ↗ to ↘ $\Rightarrow$ local max

If f goes from ↘ to ↗ $\Rightarrow$ local min.

Second D.T.

If $f''(c) > 0 \Rightarrow$ C.U. $\Rightarrow$ local min.

If $f''(c) < 0 \Rightarrow$ C.D. $\Rightarrow$ local max.

We can also use the ideas from F.D.T to find the intervals of concavity as well as inflection points.

Example: Let $f(x) = x^4 - 4x^3$.

- Find where Inc./Dec. Find local max/min
- Find where C.U./C.D. Find inflection points

$$f'(x) = 4x^3 - 12x^2 = 4x^2(x-3).$$

Critical points: $x = 0, 3$.

Inc./Dec. — neither. — local min

	$(-\infty, 0)$	$(0, 3)$	$(3, \infty)$
$f(x)$	$\searrow$	$\searrow$	$\nearrow$
$f'(x)$	<0	<0	>0

Inc: $(3, \infty)$
Dec: $(-\infty, 0) \cup (0, 3)$

Local min at $x = 3$.
Neither max/min at $x = 0$.

Clicker Q:

A: $\searrow \searrow \searrow$
B: $\searrow \searrow \nearrow$
C: $\searrow \nearrow \searrow$
D: $\nearrow \nearrow \searrow$

Concavity: $f''(x) = 12x^2 - 24x$
$= 12x(x-2)$.

Possible inflection points at $x = 0, 2$.

inflection points

	$(-\infty, 0)$	$(0, 2)$	$(2, \infty)$
$f(x)$	C.U.	C.D.	C.U.
$f''(x)$	>0.	<0	>0

Clicker Q:

A: $\cap\ \cap\ \cap$
B: $\cup\ \cap\ \cap$
C: $\cup\ \cap\ \cup$
D: $\cap\ \cap\ \cup$

A: ~~C.D. C.D. C.D~~
B:
C:

Concave up: $(-\infty, 0) \cup (2, \infty)$
Concave down: $(0, 2)$.

Inflection points at $x = 0$ and $x = 2$.

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Let's find the y-values of these points.

$$f(x) = x^4 - 4x^3.$$

$$f(0) = 0$$

$$f(2) = 2^4 - 4\cdot 2^3$$

$$= 2\cdot 2^3 - 4\cdot 2^3 = -2\cdot 2^3 = -16.$$

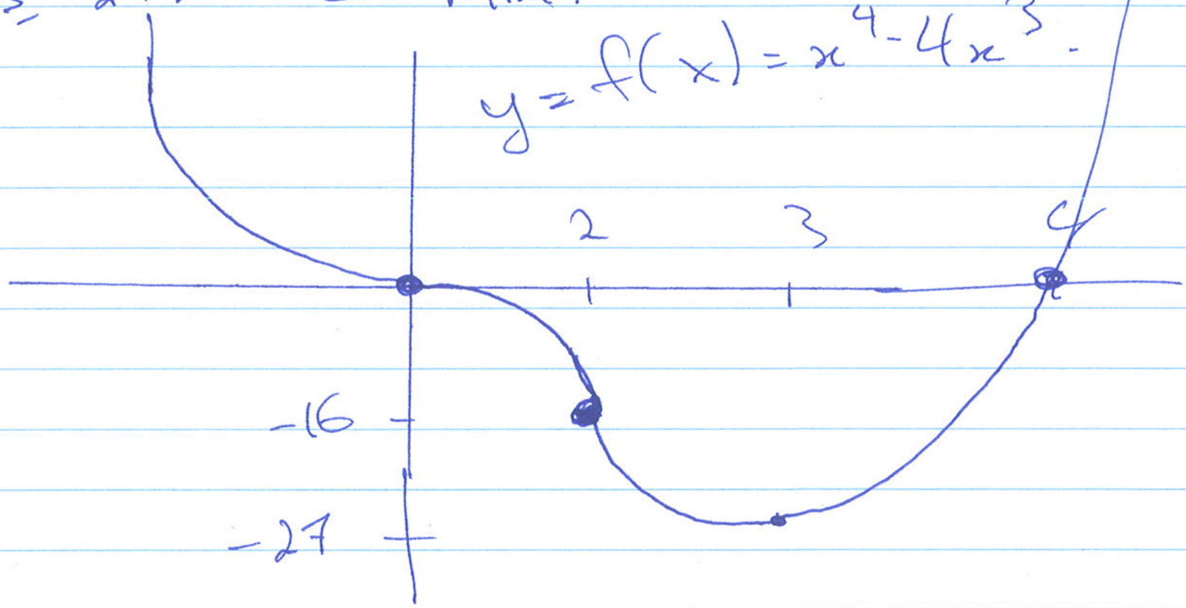
$$f(3) = 3\cdot 3^3 - 4\cdot 3^3$$

$$= -3^3 = -27.$$

$(0, 0)$: I.P.

$(2, -16)$: I.P.

$(3, -27)$: L. Min.



x-int. at $x = 4$.

$$f(x) = x^3(x - 4).$$

⑥

Sketch the graph of a function satisfying the following properties:

Dec: $(-\infty, 0) \cup (4, 6) \cup (6, \infty)$

Inc: $(0, 4)$

$f'(x)$ D.N.E. at $x = 0$ and $x = 6$.

CU: $(6, \infty)$

CD: $(-\infty, 0) \cup (0, 6)$.

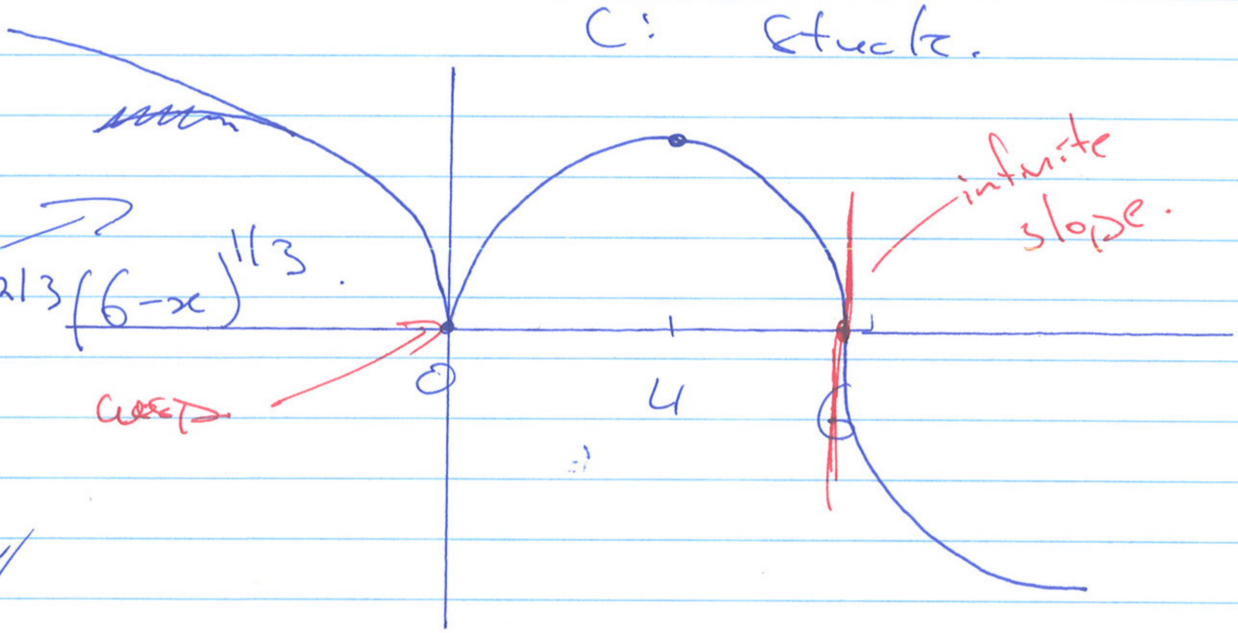
Clickers:

A: Working

B: Done

C: Stuck.

$f(x) = x^{2/3}(6-x)^{1/3}$.



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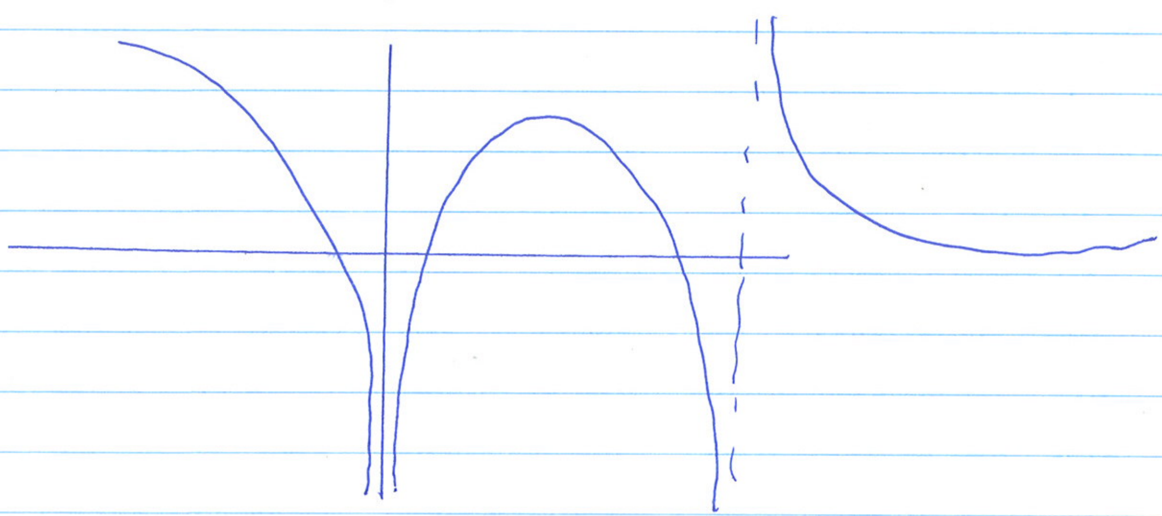

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§4.5 Summary of Curve Sketching

Use all the tools we have to sketch graphs.

Example: $f(x) = \dfrac{2x^2}{x^2-1}$.

1. Domain
2. Intercepts (if not too hard)
3. Asymptotes.
4. Inc/Dec. Local max/min
5. Concavity and Inflection Points.

1. Domain: $\{x \in \mathbb{R} : x \neq \pm 1\}$

$(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$.

2. y-int: $f(0) = 0 \Rightarrow y = 0$.

x-int: $f(x) = 0 \Rightarrow x = 0$

$(0, 0)$.

3. Horizontal: $\lim\limits_{x \to \infty} \dfrac{2x^2}{x^2-1} = \lim\limits_{x \to \infty} \dfrac{2}{1 - 1/x^2}$.

$= 2$.

$\lim\limits_{x \to -\infty} \dfrac{2x^2}{x^2-1} = 2$.

Asymptote at $y = 2$.

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Vertical: $\lim_{x \to 1^+} \frac{2x^2}{x^2-1} = \text{"}\left(\frac{+}{+}\right)\text{"} = +\infty.$

$$\lim_{x \to 1^-} \frac{2x^2}{x^2-1} = -\infty.$$

$$\lim_{x \to -1^+} \frac{2x^2}{x^2-1} = -\infty.$$

$$\lim_{x \to -1^-} \frac{2x^2}{x^2-1} = +\infty.$$

Asymptotes at $x = \pm 1$.

4. Inc/Dec. $f'(x) = \frac{-4x}{(x^2-1)^2}$.

Critical points: 0, ± 1.

asymptotes

	$(-\infty, -1)$	$(-1, 0)$	$(0, 1)$	$(1, \infty)$
$f(x)$	$\nearrow$	$\nearrow$	$\searrow$	$\searrow$
$f'(x)$	>0	>0	<0	<0

local max.

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5. Concavity: $f''(x) = \dfrac{12x^2+4}{(x^2-1)^3}$.

	$(-\infty, -1)$	$(-1, 1)$	$(1, \infty)$
$f(x)$	C.U.	C.D.	C.U.
$f''(x)$	>0	<0	>0