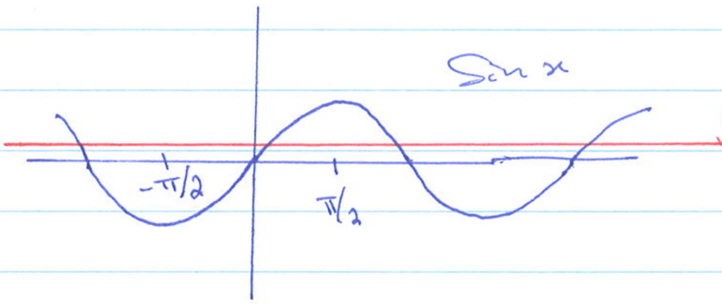
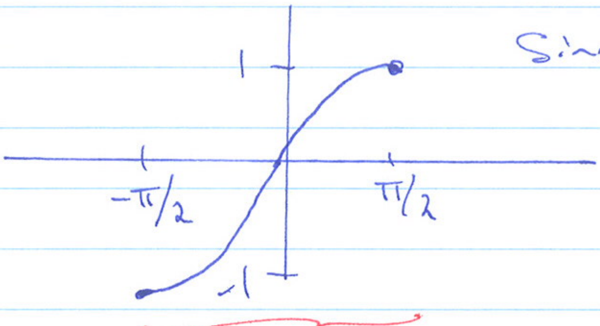
1 Jan. 29.

Inverse Trig. Functions

Where are the trig. functions invertible? (one-to-one).



fails ~~vertical~~ horizontal line test.



one-to-one on this domain.

$\sin x$ is invertible on $\{x : -\pi/2 \le x \le \pi/2\}$.

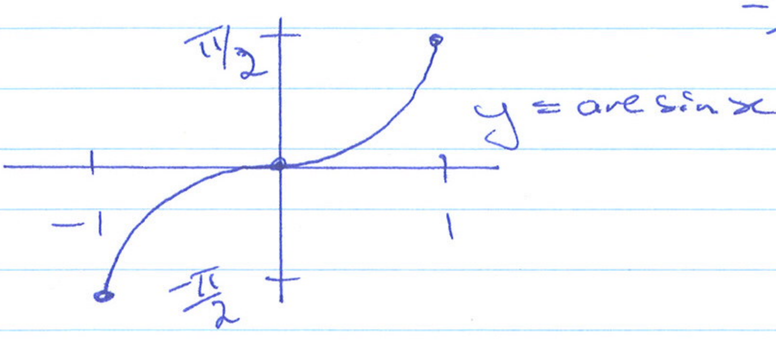
We can define the inverse sine function

$\sin^{-1} x$, $\arcsin x$.

So, when we write $y = \sin^{-1}(x)$

we mean $x = \sin(y)$

$-\frac{\pi}{2} \le y \le \pi/2$.



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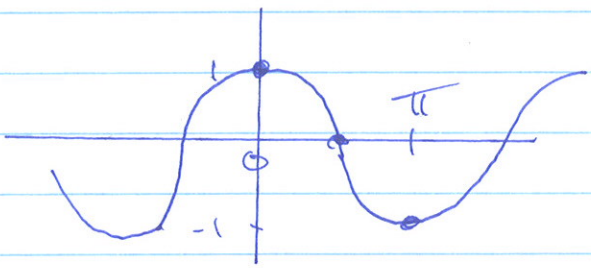
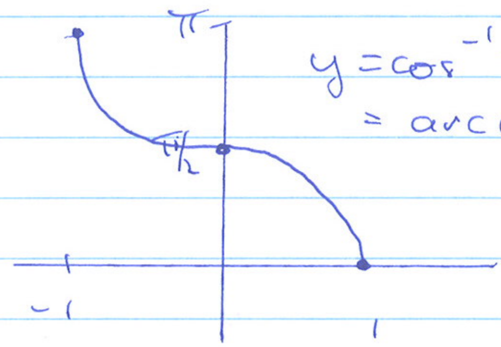
Clicker Q: On which domain is $\cos x$ invertible?

A: $\{-\pi \le x \le \pi\}$ → B: $\{0 \le x \le \pi/2\}$

→ C: $\{0 \le x \le \pi\}$ D: $\{-\pi/2 \le x \le \pi/2\}$.





$y = \cos^{-1} x$
$= \arccos x$.

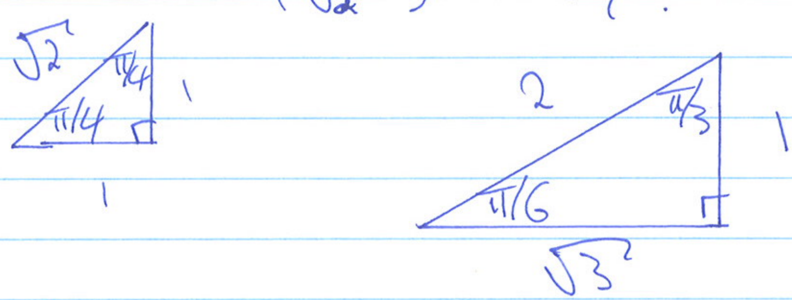
cosine is invertible on this domain

So, $y = \cos^{-1} x \iff x = \cos(y)$, $0 \le y \le \pi$.

Examples:

$$\arcsin(1) = \pi/2.$$

$$\arccos\left(\frac{1}{\sqrt{2}}\right) = \pi/4.$$



Clicker Q: $\arcsin(\sin(2\pi)) = ?$

A: 2π B: π

→ C: 0 D: 1.

$$\sin(2\pi) = 0$$
$$\arcsin(0) = 0.$$

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Clicker Q: $\sin(\arcsin(\pi))$

A: π C: 1

B: 0 $\longrightarrow$ D: None of the above.

$\arcsin(\pi)$ is undefined.

So $\sin(\arcsin(\pi))$ is undefined.

Example: Simplify $f(x) = \sin(\cos^{-1} x)$.

We want to write this sine in terms of cosine.

We need: $\sin^2 x + \cos^2 x = 1$.

$$\sin^2 x = 1 - \cos^2 x$$

$$\sin x = \pm\sqrt{1 - \cos^2 x}.$$

We want to put $y = \cos^{-1} x$ in the above.

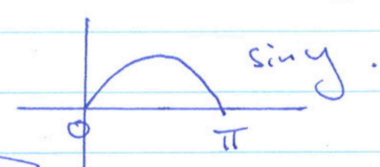
$$\sin(\cos^{-1} x) = \sin y = \pm\sqrt{1 - \cos^2 y}.$$

positive or negative root?

Well is $\sin y$ positive or negative?

Well $y = \cos^{-1} x$ and so $0 \le y \le \pi$.

and so $\sin y \geq 0$



So,
$$\sin y = +\sqrt{1 - \cos^2 y}$$
$$= \sqrt{1 - [\cos(\cos^{-1} x)]^2}$$
$$= \sqrt{1 - x^2}.$$

Hence, $\sin(\cos^{-1} x) = \sqrt{1 - x^2}$.

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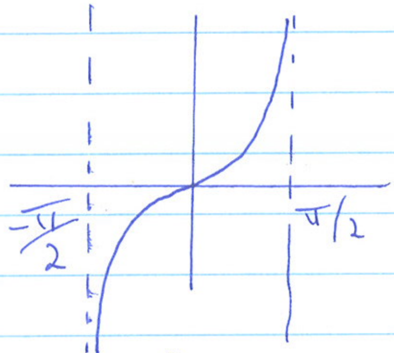
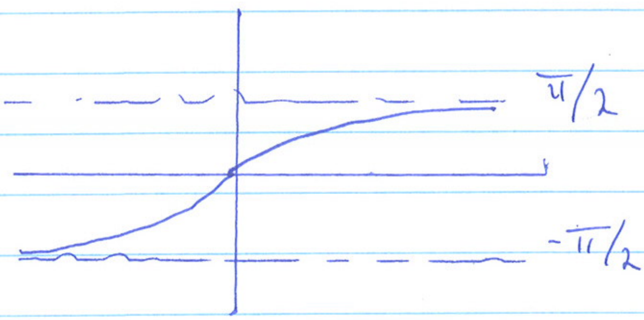
... and arctan x = $\tan^{-1} x$



$y = \tan x$
on $-\frac{\pi}{2} \le x \le \pi/2$



$y = \arctan x$
$= \tan^{-1} x$.

§ 3.5 Implicit Differentiation

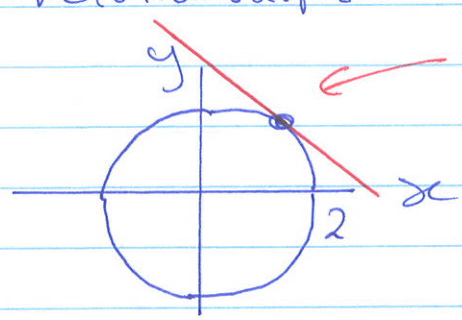
So far we have been working explicit functions such as,

$$y = x^2 - x \quad , \quad y = \sin(2x).$$

We can find tangent lines by finding the derivative $\frac{dy}{dx}$.

What about other relationships

$$x^2 + y^2 = 4.$$



find slope?

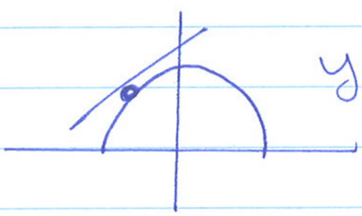
The circle is not a function, but it still has a tangent line.

In, $x^2 + y^2 = 4$, y is an implicit function of x.

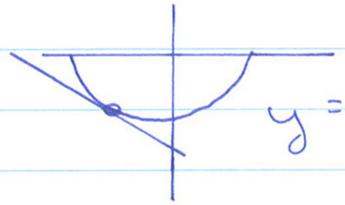
⑤ Jan. 29 $\boxed{x^2+y^2=4.}$

We could find y explicitly,

$$y^2 = 4 - x^2$$
$$y = \pm\sqrt{4-x^2}.$$



$y=\sqrt{4-x^2}$



$y=-\sqrt{4-x^2}$

We can then take the derivative.

Top: $y=(4-x^2)^{1/2}$

$$\frac{dy}{dx} = y' = \frac{1}{2}(4-x^2)^{-1/2}\cdot(-2x)$$
$$= \frac{-x}{\sqrt{4-x^2}} = \frac{-x}{y}.$$

Bottom: $y=-(4-x^2)^{1/2}$

$$\frac{dy}{dx} = y' = +\left(\frac{-x}{-\sqrt{4-x^2}}\right) = \frac{-x}{y}.$$

Instead of solving for y and taking the derivative of each piece, we can differentiate implicitly.

Write $x^2+y^2=4$ (We want $\frac{dy}{dx}=y'$).

Think of y as a function of x and differentiate both sides using Chain Rule.

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$$x^2 + y^2 = 4$$

$$\frac{d}{dx}\left(x^2+y^2\right) = \frac{d}{dx}(4)$$

$$\frac{d}{dx}(x^2) + \frac{d}{dx}(y^2) = 0$$

$$2x + 2y \cdot \frac{dy}{dx} = 0$$

~~aside: pretend~~ $\cancel{y = \sqrt{1-x^2}}$

$$\cancel{\frac{d}{dx}(y) = \frac{d}{dx}\left(\sqrt{1-x^2}\right) = \frac{1}{2}(1-x^2)^{-1/2}\cdot(-2x)}$$

$$\cancel{= \frac{1}{2}y^{-1/2}\cdot\frac{dy}{dx}}$$

Pretend $y = \sin x$.

$$\frac{d}{dx}(y^2) = \frac{d}{dx}\left((\sin x)^2\right) = 2\sin x \frac{d}{dx}(\sin x)$$

$$= 2y \cdot \frac{dy}{dx}.$$

Pretend: $y = y(x)$

$$\boxed{\frac{d}{dx}\left(y^2(x)\right) = 2y(x)\cdot y'(x)}$$

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Examples: Find • $\frac{d}{dx}(y) = \frac{dy}{dx} = y'$.

• $\frac{d}{dx}(x \cdot y) = \frac{d}{dx}(x) \cdot y + x \frac{d}{dx}(y)$

$= 1 \cdot y + x \frac{dy}{dx}$

$= y + x\frac{dy}{dx}$.

You can also use y'.

• $(x \cdot y)' = x' y + x y'$
$= y + xy'$.

• $\frac{d}{dx}(y^2) = 2y \cdot y'$.

$\left(y = x \Rightarrow \frac{dy}{dx} = 1 \right)$

Back to the circle.

$$x^2 + y^2 = 4$$

take d/dx of both sides

$$\frac{d}{dx}(x^2 + y^2) = \frac{d}{dx}(4)$$

$$2x + 2y\frac{dy}{dx} = 0.$$

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And solve for $\frac{dy}{dx}$.

$$2y \frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = \frac{-2x}{2y} = \boxed{\frac{-x}{y}}.$$