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Midterm: Feb. 3 in class (60 min)
8am - 9am.

Will cover everything we have done so far (including Chain Rule (today)).

Limits:
- conceptual Qs
- Computational Qs
- asymptotes

Continuity:
- find discontinuity / make $f(x)$ continuous etc.
- IVT

Derivatives:
- conceptual Qs (differentiability)
- limit definition
- tangent lines
- power/product/quotien/chain rules
- trig.

When using a theorem (IVT, Squeeze thm)
- State the $\underbrace{}_{\text{name of the}}$ theorem you are using.
- verify that the conditions are met.

You may need to evaluate something like:
$\sin(\pi/2)$, $\tan(\pi/3)$

(unit circle / special triangles).

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Additional office Hours Mon/Fri ?

Links to 10 previous Midterms on Webpage.

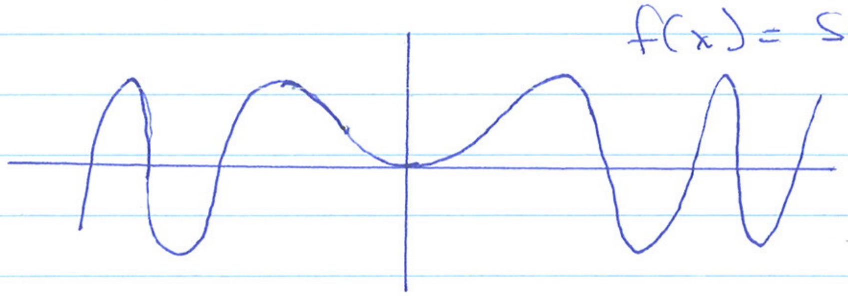
Friday: A : 10am
B: 11am
C: 12 noon
D: 1pm
E: 2pm

12 – 1:30pm

Monday: 10 – 11:30 $\leftrightarrow$ in my office AA 137.

§ 3.4 Chain Rule

$f(x) = \sin(x^2)$



Clicker Q: What is $f'(x)$.

~~A~~: $\cos(x^2)$ C: $2x\cos(2x)$

~~B~~: $\cos(2x)$ $\longrightarrow$ D: $2x\cos(x^2)$.

Chain Rule: "take the derivative of the outside (leave the inside the same) and multiply by the derivative of the inside".

Let $h(x) = f(g(x))$ then

$$\boxed{\begin{array}{l} h'(x) = f'(g(x)) \cdot g'(x). \\ \quad \text{or if } y = f(u),\ u = g(x) \\ \dfrac{dy}{dx} = \dfrac{dy}{du} \cdot \dfrac{du}{dx}. \end{array}}$$

Example: $f(x) = \sin(x^2)$

(1): Let $f(u) = \sin u$, $f'(u) = \cos u$
$u = g(x) = x^2$; $g'(x) = 2x$.

$$h'(x) = f'(g(x))\, g'(x)$$
$$= \cos(x^2) \cdot 2x.$$

(2): $$\frac{d}{dx} h(x) = \cos(x^2) \cdot \frac{d}{dx}(x^2)$$
$$= 2x\cos(x^2).$$

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Examples: Take these derivatives

1. $h(x) = e^{2x}$ | A: 0 done
2. $h(x) = \sqrt{x^2+1} = (x^2+1)^{1/2}$ | B: 1
3. $h(x) = (g(x))^n$ | C: 2
4. $h(x) = e^{\cos(x^2)}$ | D: 3
 | E: 4.

5. $\tan(\sqrt{x^2+1})$

1. $h'(x) = 2e^{2x}$.

2. $h'(x) = \frac{1}{2}(x^2+1)^{-1/2} \cdot 2x$.

$h(x) = f(g(x))$, $f(u) = u^{1/2}$

$u = g(x)$ x^2+1.

$f'(u) = \frac{1}{2}u^{-1/2}$

3. $n\{g(x)\}^{n-1} g'(x)$

4. $\boxed{h'(x) = \cancel{-\cos(x^2)}\, e^{\cos(x^2)} \cdot 2x \sin(x^2)}$

$h'(x) = e^{\cos(x^2)} \cdot \frac{d}{dx}(\cos(x^2))$

$= -e^{\cos(x^2)} \sin(x^2) \cdot 2x$.

$h(x) = e^{\cos(x^2)}$

$f(u) = e^u$

$u = g(x) = \cos(x^2)$

5. $h(x) = \tan\left(\sqrt{x^2+1}\right)$.

$$h'(x) = \sec^2\left(\sqrt{x^2+1}\right) \cdot \frac{d}{dx}\left(\sqrt{x^2+1}\right)$$

$$= \sec^2\left(\sqrt{x^2+1}\right) \cdot \frac{1}{2}\left(x^2+1\right)^{-1/2} \cdot \frac{d}{dx}\left(x^2+1\right)$$

$$= \sec^2\left(\sqrt{x^2+1}\right) \cdot \frac{1}{2}\left(x^2+1\right)^{-1/2} \cdot 2x \; //.$$

HW: $\sqrt{x + \sqrt{x + \sqrt{x}}}$.

End of Midterm Testable Material

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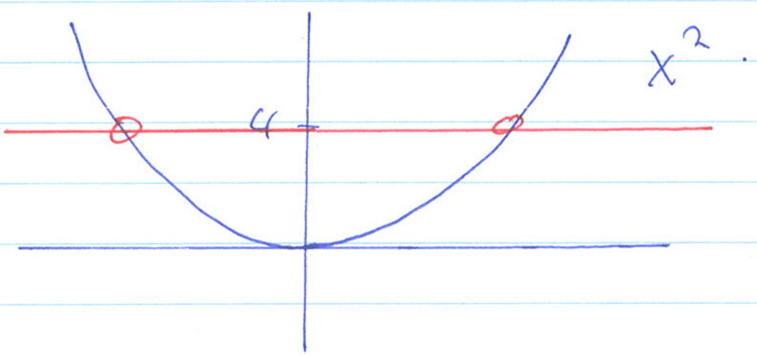
§1.6 Inverse Functions and Logarithms

Sometimes we may want to undo a function.

Examples: $4 = x^2$, $2 = e^x$, $1/2 = \sin(x)$ } Solve for x.

For $4 = x^2$ we can take the square root.

$\pm\sqrt{4} = x$; but we get two answers.



$y = x^2$ fails the horizontal line test.

$y = x^2$ is not one-to-one.

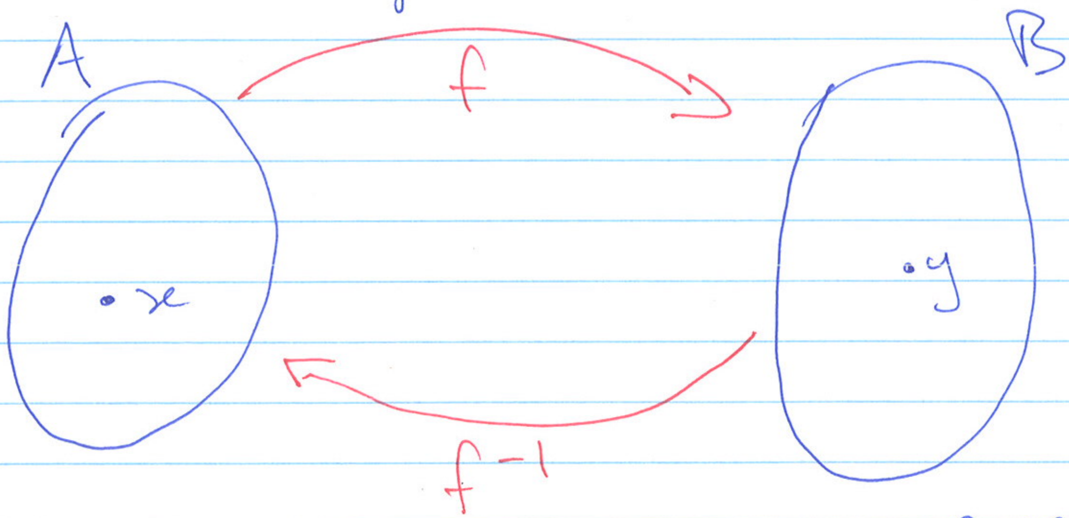
Definition: A function is one-to-one (injective) if it never takes the same y-value more than once. That is,

if $x_1 \neq x_2$ then $f(x_1) \neq f(x_2)$.

Def: (Horizontal Line test p.60)

A function is one-to-one if and only if no horizontal line intersects the graph $y = f(x)$ more than once.

If we have a one-to-one function say f, with domain A and range B we can define the inverse function, called $f^{-1}(x)$, which has domain B and range A.

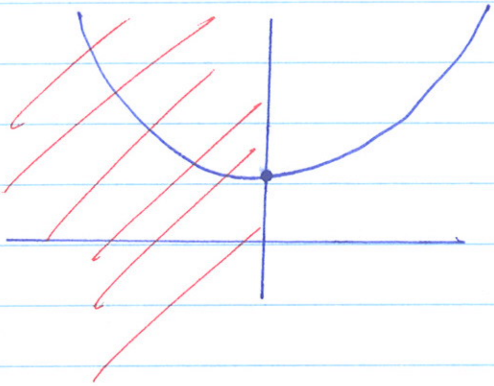


So, if $y = f(x)$ then $f^{-1}(y) = x$.

We also have
$$f^{-1}(f(x)) = x$$
$$f(f^{-1}(y)) = y.$$

[Note $f^{-1}(x) \neq \dfrac{1}{f(x)}$].

Examples: $f(x) = x^2+1$. Find Domain/Range
Find inverse.



is one-to-one on the domain $\{x \geq 0\}$ and has range $\{y \geq 1\}$.

To find the inverse let $y = x^2+1$ and solve for x.

$$y - 1 = x^2$$
$$\sqrt{y-1} = x$$

and switch x and y.

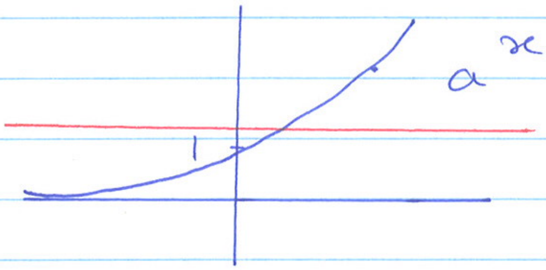
$$y = f^{-1}(x) = \sqrt{x-1}$$

has domain : $\{x \geq 1\}$
and range : $\{x \geq 0\}$.

You can verify that $f^{-1}(f(x)) = x$.

Logarithmic Functions.

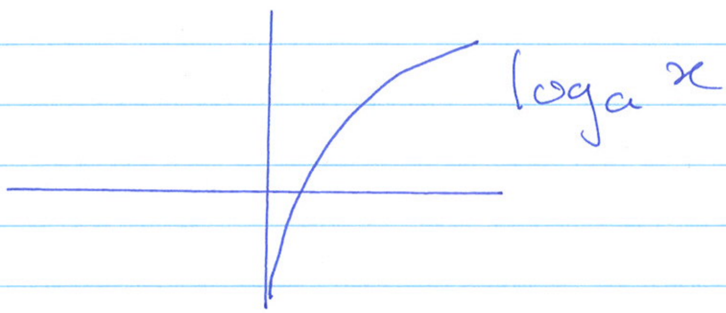
Consider the Exponential Function.



$f(x) = a^x$, $a > 0$.

this function is one-to-one
(horizontal line test)

The inverse is the function $\log_a x$.



So, $y = \log_a x \iff x = a^y$.

and $\log_a(a^x) = x$, $a^{\log_a(x)} = x$.

We have some logarithm rules.

$$\begin{cases} \log_a(xy) = \log_a(x) + \log_a(y). \\ \log_a(x/y) = \log_a(x) - \log_a(y) \\ \log_a(x^r) = r\log_a(x). \end{cases}$$

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The most important is logorithms is

$$\ln x = \log_e x .$$

Change of Base: $\log_a(x) = \dfrac{\ln x}{\ln a}$:

$$\log_2(7) = \frac{\ln(7)}{\ln(2)} .$$

Last class $h(x) = a^x$.

Found $(2^x)' = 2^x(0.693\ldots)$

& Write $a = e^{\ln a}$

$$h(x) = a^x = \left(e^{\ln a}\right)^x = e^{\ln a \cdot x} .$$

$$h'(x) = e^{\ln a \cdot x} \frac{d}{dx}(\ln a \cdot x) = e^{\ln a \cdot x} \ln a$$

$$= \left(e^{\ln a}\right)^x \ln a$$

$$= a^x \ln a .$$

chain rule.