MLC opens today.

Midterm 1: Feb. 3 in class.

Intermediate Value Theorem (IVT)

Last class we tried to find where

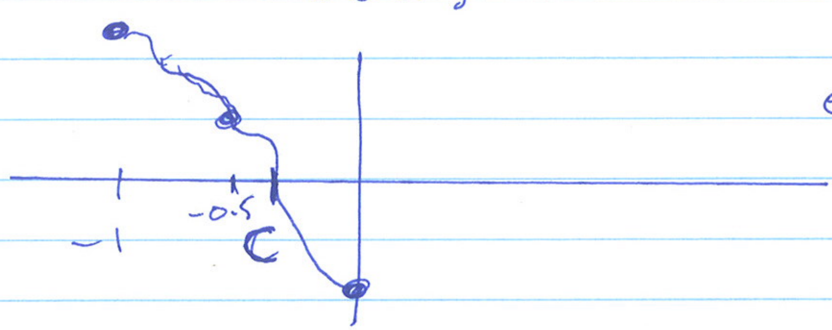
$$f(x) = e^{-x} - 2(x+1) = 0.$$

We checked

$$f(-1) = e^{1} > 0$$
$$f(0) = -1 < 0.$$

Since the function is continuous it must cross the x-axis.
By the IVT there exists c between -1 and 0 where

$$f(c) = 0.$$

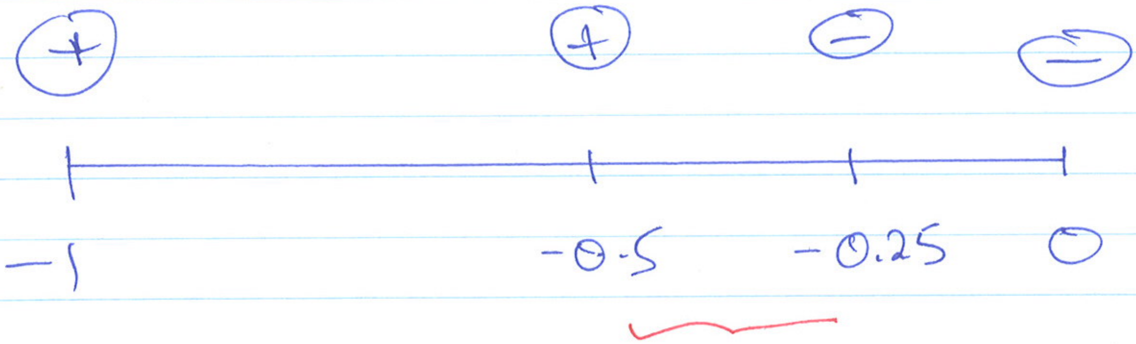


~~$c \in (-1,0)$~~

$c \in (-1,0)$

what if $f(-0.5) = 0.6487\ldots > 0$

$c \in (-0.5, 0)$

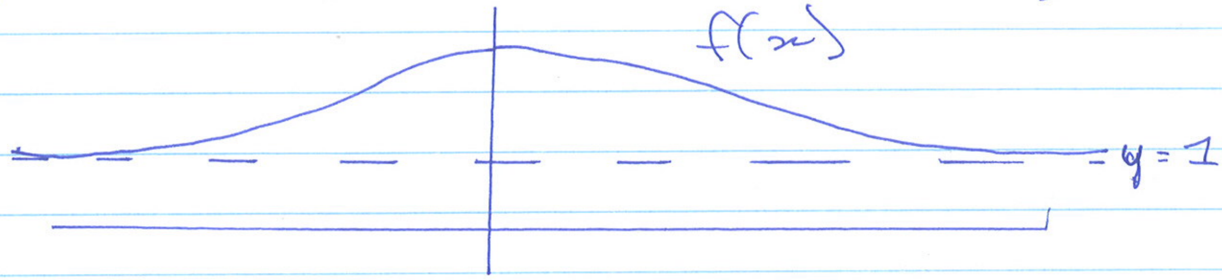


c is in here somewhere.

The answer is about $c = -0.3149\ldots$

Called the Bisection Method.

§ 2.6 Limits at Infinity



We write $\lim_{x \to \infty} f(x) = 1$

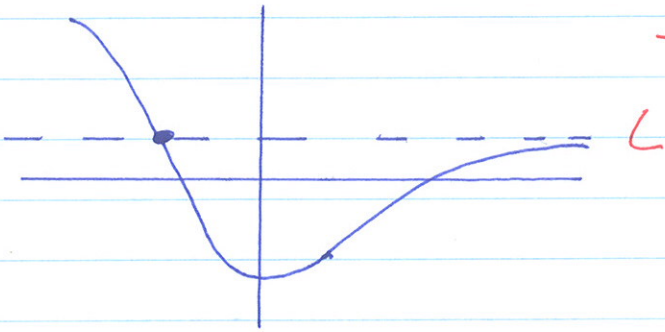
and $\lim_{x \to -\infty} f(x) = 1$.

As x gets large, $f(x)$ gets close to 1.

If $\lim_{x \to \infty} f(x) = L$ or $\lim_{x \to -\infty} f(x) = L$

we say $f(x)$ has a horizontal asymptote at $y = L$.

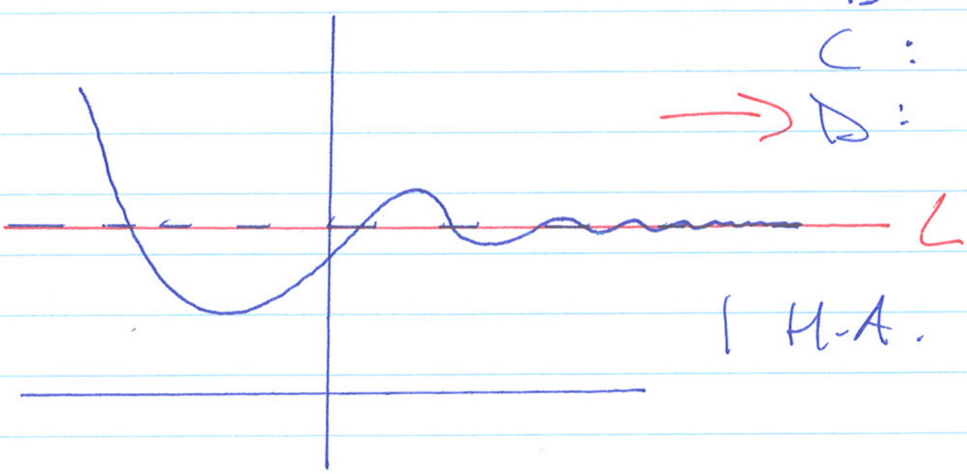
Clicker Q: Can a function cross its horizontal asymptote?



A = Yes
B = No

Clicker Q: How many times can a function cross its horizontal asymptote?

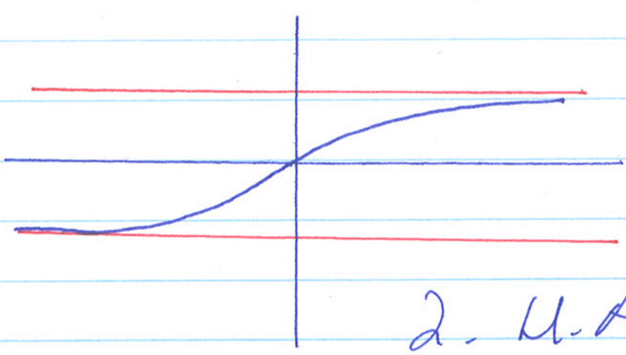
A = 1
B = 2
C : A few times
D : infinitely many times.



1 H.A.

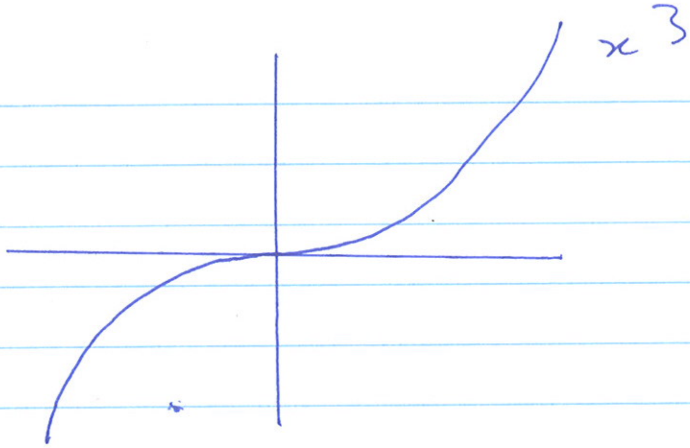
Clicker Q! How many H.A. can a function have?

A : 0
B : 1
C : 2
D : 3
E : Infinitely many.



$\tan^{-1} x$ = arctan x

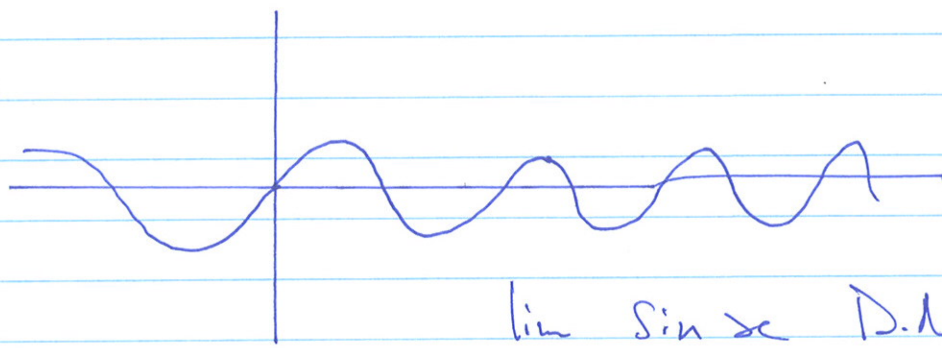
2 H.A.



zero H.A.

<u>Q:</u> $\sin x$

no H.A.



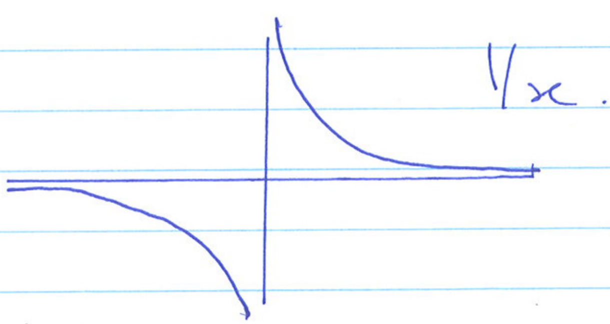
$\lim_{x \to \infty} \sin x$ D.N.E.

To find a functions H.A. (if any) compute

$$\lim_{x \to \infty} f(x) \quad \text{and} \quad \lim_{x \to -\infty} f(x).$$

<u>Example:</u> $f(x) = 1/x$.

$$\lim_{x \to \infty} \frac{1}{x} = 0, \quad \lim_{x \to -\infty} \frac{1}{x} = 0.$$



H.A. at $y = 0$.

Example: $f(x) = \dfrac{-x^2+7}{2x^2+5x}$.

$$\lim_{x\to\infty} \frac{-x^2+7}{2x^2+5x} = \lim_{x\to\infty} \frac{x^2(-1+7/x^2)}{x^2(2+5x/x^2)}$$

$$= \lim_{x\to\infty} \frac{-1+7/x^2}{2+5/x}$$

Small

$$= -1/2.$$

Example: $\displaystyle\lim_{x\to\infty} \frac{-x^2+7}{2x^3+5x} = \lim_{x\to\infty} \frac{x^3(-1/x+7/x^3)}{x^3(2+5/x^2)}$

$$= \lim_{x\to\infty} \frac{-1/x+7/x^3}{2+5/x^2}.$$

$$= 0/2 = 0.$$

Example: $f(x) = \dfrac{\sqrt{4x^2+3}}{x-7}$.

$$\lim_{x\to\infty} f(x) = \frac{\sqrt{x^2(4+3/x^2)}}{x-7}$$

7

$$= \lim_{x\to\infty} \frac{\sqrt{x^2}\sqrt{4+3/x^2}}{x-7}, \quad x \geq 0.$$

$$= \lim_{x\to\infty} \frac{x\sqrt{4+3/x^2}}{x-7} = \lim_{x\to\infty} \frac{\cancel{x}\sqrt{4+3/x^2}}{\cancel{x}(1-7/x)}.$$

$$= \lim_{x\to\infty} \frac{\sqrt{4+3/x^2}}{1-7/x}.$$

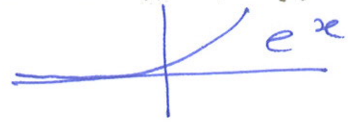
$$= \frac{\sqrt{4}}{1} = 2.$$

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{\sqrt{x^2}\sqrt{4+3/x^2}}{x-7}.$$

$$|x| = \sqrt{x^2} = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

$$= \lim_{x\to-\infty} \frac{-x\sqrt{4+3/x^2}}{x-7} = \lim_{x\to-\infty} \frac{-x\sqrt{4+3/x^2}}{x(1-7/x)}$$

$$= -2.$$



<u>Example:</u> $\lim\limits_{x\to\infty} \underbrace{e^{-x}}_{\to 0} + x^{5/2} - x^2$.

$$= \lim_{x\to\infty} x^{5/2} - x^2 \cancel{=} \underbrace{\infty - \infty}_{\text{meaningless.}}$$

$$= \lim_{x\to\infty} x^{5/2}\left(1 - x^2/x^{5/2}\right)$$

$$= \lim_{x\to\infty} x^{5/2}\left(1 - 1/x^{1/2}\right) .$$

$$= \left(\lim_{x\to\infty} x^{5/2}\right)\left(\lim_{x\to\infty}\left(1 - 1/x^{1/2}\right)\right)$$

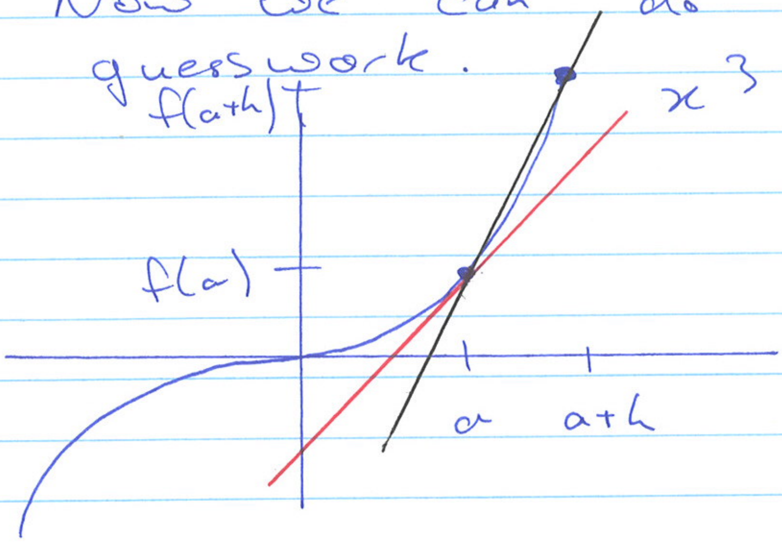
$$= \lim_{x\to\infty} x^{5/2} = \infty .$$

⑨

§ 2.7 Derivatives and Rates of Change.

Previously we approximated the slope of a tangent line by secant lines.

Now we can do this with no guesswork.



$$m_{sec} = \frac{f(a+h) - f(a)}{a+h-a} = \frac{f(a+h) - f(a)}{h}.$$

When we take the limit as $h \to 0$ we get the slope of the tangent line.

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}.$$

this is called the derivative of $f(x)$ at the point $x = a$.

Example: Find the slope of the line tangent to $f(x) = x^2$ at the point $a = 2$.

$$f'(2) = \lim_{h \to 0} \frac{(2+h)^2 - 2^2}{h}.$$

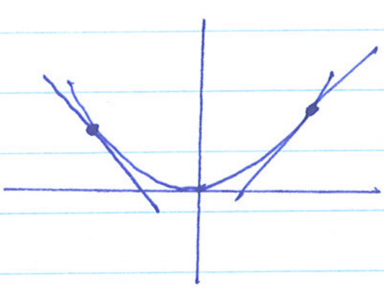
$$= \lim_{h \to 0} \frac{4 + 4h + h^2 - 4}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(4+h)}{\cancel{h}} = 4.$$

We can do this at any point $x = a$.

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}.$$

$$= \lim_{h \to 0} \frac{(a+h)^2 - a^2}{h}$$



$$= \lim_{h \to 0} \frac{\cancel{a^2} + 2ah + h^2 - \cancel{a^2}}{h}.$$

$$= \lim_{h \to 0} \frac{\cancel{h}(2a+h)}{\cancel{h}} = 2a.$$

a function.