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§ 3.7 Rates of Change in Science.

Often in science we are interested in how a quantity is changing. The derivative of a function is the rate of change.

Suppose the position of a particle is given by

$$s(t) = t^3 - 6t^2 + 9t, \quad t \geq 0.$$

The velocity: $v(t) = s'(t)$

The acceleration: $a(t) = v'(t) = s''(t)$.

Clicker-Q: When is the particle moving forward?

A: $0 < t < 1$ and $t > 3$ B: $1 < t < 3$

C: $0 < t < 1$ and $2 < t < 3$ D: $1 < t < 2$, $t > 3$

E: $t > 2$.

Look for $v(t) > 0$.

$$s'(t) = v(t) = 3t^2 - 12t + 9 = 3(t^2 - 4t + 3)$$
$$= 3(t-1)(t-3).$$

$t < 1$: $v(t) > 0$

$1 < t < 3$: $v(t) < 0$

$t > 3$: $v(t) > 0$

moving forward.

Clicker Q: When is the particle speeding up?

Same A – E as before.

$v(t) = 3(t-1)(t-3)$

$a(t) = 6t - 12 = 6(t-2)$.

$t < 2$: $a(t) < 0$

$t > 2$: $a(t) > 0$.

$0<t<1$	$1<t<2$	$2<t<3$	$t>3$
$v(t) > 0$	$v(t) < 0$	$v(t) < 0$	$v(t) > 0$
$a(t) < 0$	$a(t) < 0$	$a(t) > 0$	$a(t) > 0$
slowing down	speeding up	slowing down	speeding up.

Answer: D.

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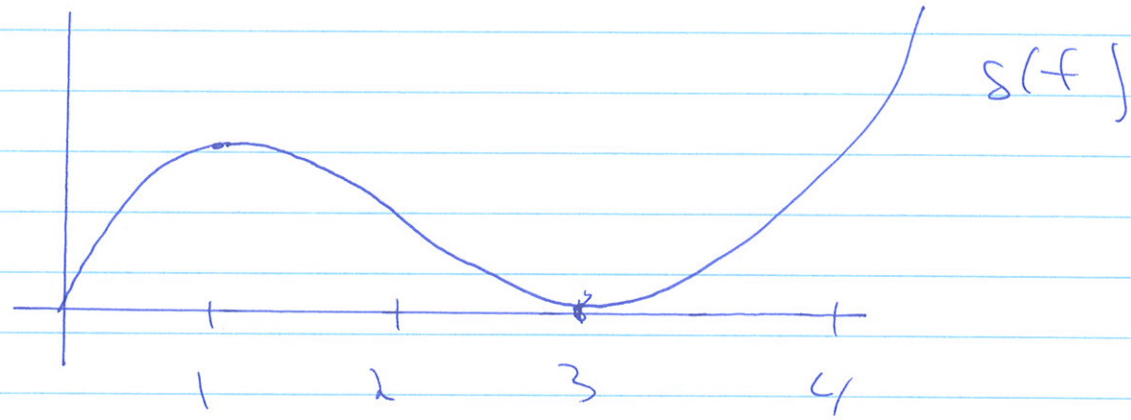

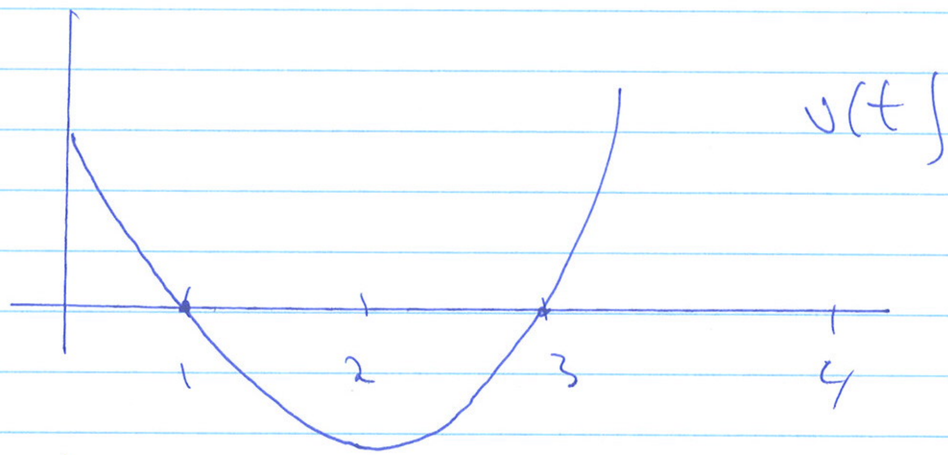

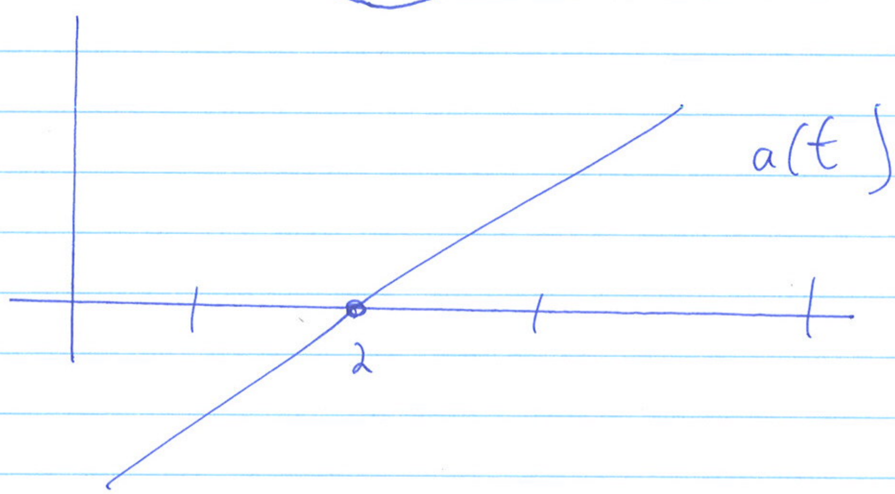

In the above $v = \frac{ds}{dt}$.

From $\left(v_{ave} = \frac{\Delta s}{\Delta t} \quad \text{take} \lim_{\Delta t \to 0} \right)$

Similarly in Electricity:

$I_{ave} = \frac{\Delta Q}{\Delta t}$, $\text{current} = \frac{\text{change in charge}}{\text{change in time}}$.

Instantaneous current is the derivative of Q with respect to time.

$$I = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}.$$

Thermodynamics: Consider a balloon of gas.

V - Volume
P - Pressure
T - Temperature.

$\uparrow P$ means $\downarrow V$ so $\frac{dV}{dP} < 0$.

A sensible measurement is "isothermal compressibility".

$$\beta = -\frac{1}{V} \frac{dV}{dP}.$$

If you had an expression for V as a function of P you could take the derivative and find B.

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Predator - Prey Model

$C(t)$ - Caribou $\qquad W(t)$ — tundra Wolves.

The interaction of the populations can be described by:

$$\begin{cases} \dfrac{dC}{dt} = aC - bCW \\ \dfrac{dW}{dt} = -cW + dCW \end{cases}$$

a, b, c, d are constants

Can the two species live in equilibrium?

This will happen if both populations are not changing.

$$\frac{dC}{dt} = \frac{dW}{dt} = 0.$$

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$$0 = aC - bCW$$
$$0 = -cW + dCW$$

$\to W = a/b$

$\to C = c/d$.

Values:
$a = 0.05$
$b = 0.001$
$c = 0.05$
$d = 0.0001$

$C = 500$
$W = 50$.

The big question is whether the equilibrium is stable or not.

§3.8 Exponential Growth and Decay

There are systems where the rate of change of a quantity is proportional to the amount of the quantity itself.

Call the quantity $y(t)$.

So, (☆) $\dfrac{dy}{dt} = ky$, k is a constant.

(population (bacteria), temperature, radioactive decay, spread of a virus.)

(☆) is the simplest differential equation.