① Apr 9

Webwork #12 due Monday.
→ additional survey questions added.

Workshops grades posted.

Exam office Hours:

all in LSK 300.

Mon 20th	:	11am - 1pm
Tues. 21st	:	3pm - 5pm
Wed. 22nd	:	12noon - 2pm.

Monday 13th: Review (post details).

MER Wiki

Limits:
- Substitution
- mult. conjugate
- common denominator.
- one-sided limits.
- Squeeze thm
- L'Hospital.

Continuity: f is continuous at $x = a$ if

$$\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = f(a).$$

IVT: guess points.

Derivatives:
- Limit Definition.
- Power/Product/Quotient/Chain
- Implicit /Log. Diff.

Functions:
- trig/ trig inverse/exp./log

Exponential growth/ decay/ Newton's Law of Cooling.

$$\frac{dP}{dt} = kP \Rightarrow P(t) = P(0)\,e^{kt}.$$

$$T(t) - T_S = (T(0) - T_S)\,e^{kt}.$$

Related Rates:
- Word problems
- picture.
- Similar triangles, trig,
- use implicit diff.

Linear Approx. / Taylor Poly.. / Remainder.

$$T_n(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \dots$$

$$|R_n(x)| \le \frac{M|x-a|^{n+1}}{(n+1)!}$$

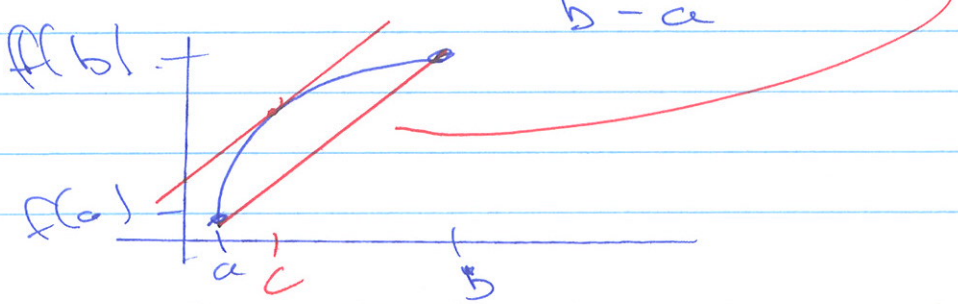
where $|f^{(n+1)}(c)| \le M$ for c between x and a.

Abs Max/Min: Closed Interval Method.

Local Max/min:
- First Derivative Test
- Second Derivative Test.

Mean Value Theorem / Rolle's Theorem.
- Show f has an exact # of sol.

$$f'(c) = \frac{f(b) - f(a)}{b-a}$$



Curve Sketching:

1. Domain
2. Intercepts.
3. Asymptotes.

- vertical: $x = a$: $\lim_{x \to a^-} f(x) = \pm\infty$

  $\lim_{x \to a^+} f(x) = \pm\infty$.

- horizontal: $\lim_{x \to \pm\infty} f(x) = ?$

- Slant asymptotes:

Want: $\lim_{x \to \pm\infty} f(x) - (mx + b) = 0$.

$$\frac{x^3 + x + 1}{x^2 + 4}.$$

4. Intervals Inc / Dec.
   Local Max Min.

5. Concavity / Inflection points

OR/ Sketch a graph with the following properties.

Optimization: Draw Picture
Find Equation(s).
Find Eq. for the thing you want to optimize (as a function of one variable).
Use - Closed Interval Method
- First Derivative Test.

L'Hôpitals Rule "$\frac{0}{0}$" or "$\frac{\infty}{\infty}$".
If it doesn't look like you need to make it look like this.

Newton's Method: $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$.

Given x_1 find x_2.
Good/Bad choice for x_1.

Antiderivatives: • Work backwards
• Solve physics problems.

(6)

Show that $f(x) = \sqrt{x^2+4x}$ has slant asymptotes $\begin{cases} y = x+2 \\ y = -x-2. \end{cases}$

$$\lim_{x\to\infty}\left[\sqrt{x^2+4x} - (x+2)\right] \overset{\text{want}}{=} 0.$$

$$\sqrt{x^2+4x} - (x+2) \cdot \frac{\sqrt{x^2+4x}+(x+2)}{\sqrt{x^2+4x}+(x+2)}.$$

$$= \frac{x^2+4x - (x+2)^2}{\sqrt{x^2+4x}+(x+2)}$$

$$= \frac{\cancel{x^2}+\cancel{4x}-\cancel{x^2}-\cancel{4x}-4}{\sqrt{x^2+4x}+(x+2)}$$

$$= \frac{-4}{\sqrt{x^2+4x}+(x+2)}$$

want $+\infty$ rather than $-\infty$.

$$\lim_{x\to\infty}\frac{-4}{\sqrt{x^2+4x}+(x+2)} = 0.$$

$\Longrightarrow$ $y = x+2$ is slant asymptote.

Also, $\lim_{x \to -\infty} \sqrt{x^2+4x} - (-x-2)$

$$= \lim_{x \to -\infty} \sqrt{x^2+4x} + (x+2) \cdot \frac{\sqrt{x^2+4x} - (x+2)}{\sqrt{x^2+4x} - (x+2)}$$

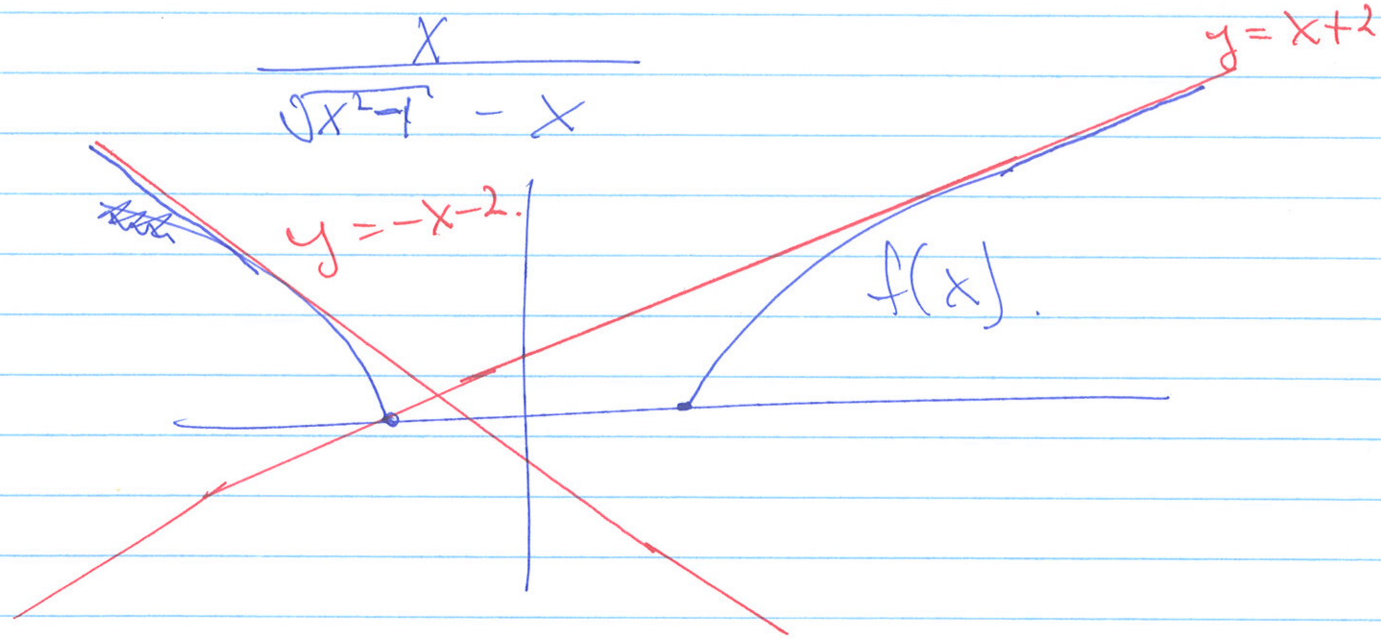
$$= \lim_{x \to -\infty} \frac{x^2+4x - (x+2)^2}{\sqrt{x^2+4x} - (x+2)}.$$

$$= \lim_{x \to -\infty} \frac{x^2+4x - x^2 - 4x - 4}{\sqrt{x^2+4x} - (x+2)}.$$

$$= \lim_{x \to -\infty} \frac{-4}{\sqrt{x^2+4x} - (x+2)} = 0.$$

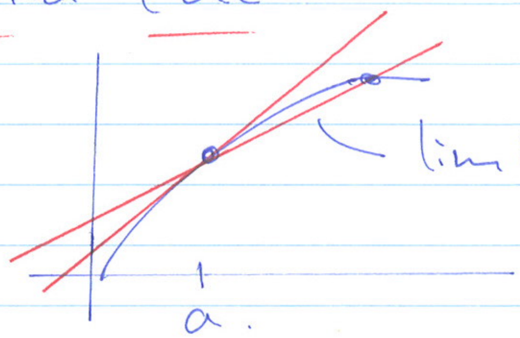
$\Rightarrow$ $y = -x-2$ is a slant asymptote

$$\frac{x}{\sqrt{x^2-1} - x}$$



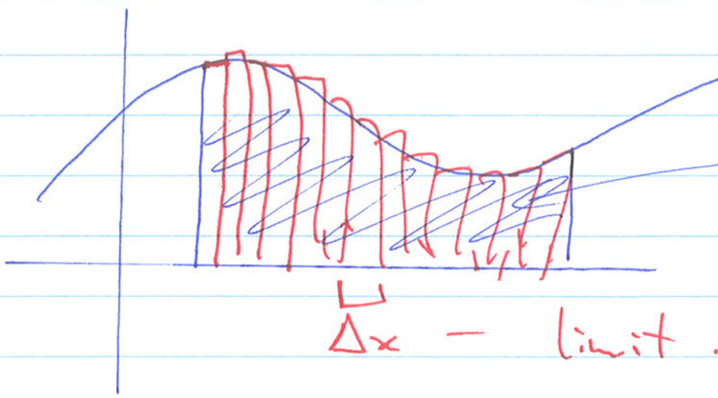
What does calc. turn into?

Differential Calc.



limit of secant.

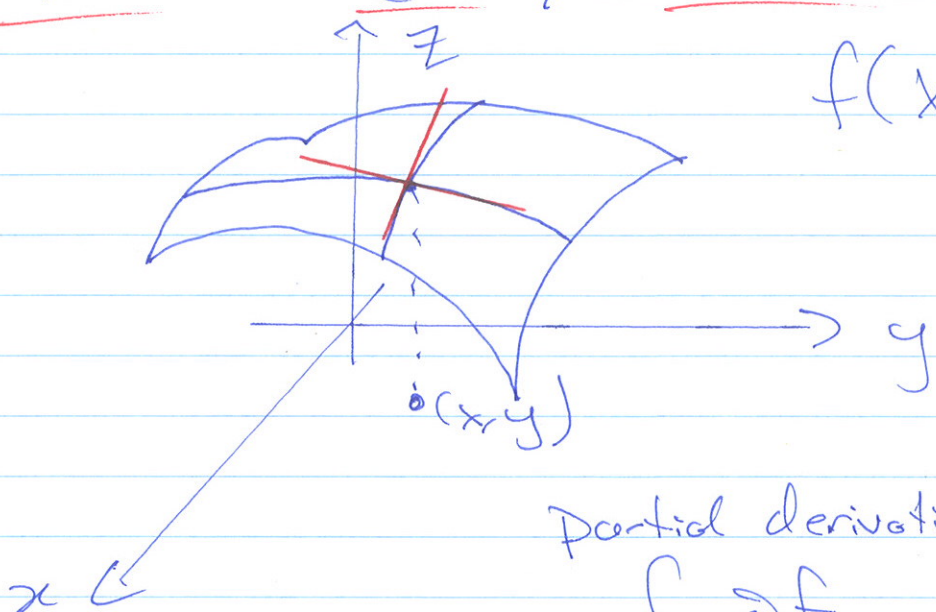
Integral Calc. Find area under a curve.



f(x).

antiderivatives.

Multivariable Calc / Vector Calc.



f(x,y).

Partial derivatives:

$$\left\{ \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right.$$

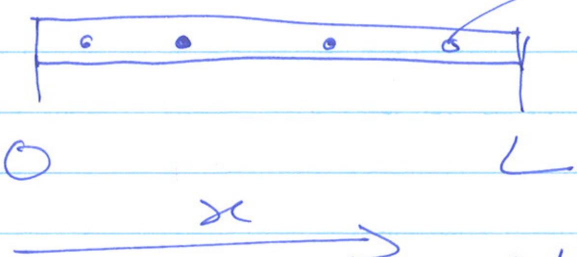
Differential Equations.

- $y' = ky$
- $y'' + 2y' + y = 0$.

Partial Differential Equations.

heat in a rod.

temperature of this point depends on time.



$u(x,t)$.

heat equation.

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \text{ \&.}$$

I study the Nonlinear Schrödinger Equation.

linear: $$i\frac{\partial \psi}{\partial t} = -\frac{\partial^2 \psi}{\partial x^2}, \quad i = \sqrt{-1}.$$

nonlinear: $$i\frac{\partial \psi}{\partial t} = -\frac{\partial^2 \psi}{\partial x^2} \pm |\psi|^2 \psi.$$