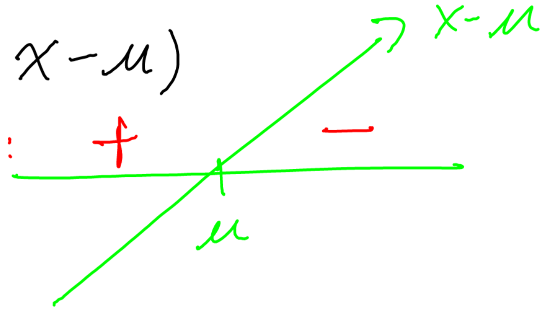
2. $N = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$

$$N'(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} \cdot \left(\frac{-\cancel{2}(x-\mu)}{\cancel{2}\sigma^2}\right)$$

$$= \underbrace{\frac{-1}{\sigma^3\sqrt{2\pi}}}_{-}\ \underbrace{e^{\frac{-(x-\mu)^2}{2\sigma^2}}}_{\text{always } +} \cdot (x-\mu)$$



(c) $\therefore$ N is increasing for $x<\mu$ and decreasing for $x>\mu$. This forces $N(\mu)$ to be a global maximum. There is no minimum value since e^{u} gets close to 0, but is never actually equal to zero. i.e. $\lim_{x\to\pm\infty} N(x) = 0$.

(a) The range is $(0, N(\mu)) = (0, 1/\sigma\sqrt{2\pi})$

(b) $\left(\mu, \frac{1}{\sigma\sqrt{2\pi}}\right)$ is a global maximum.

(d) We need the 2nd derivative.

$$N'(x) = \frac{-1}{\sigma^3\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} \cdot (x-\mu)$$

$$N''(x) = \frac{-1}{\sigma^3\sqrt{2\pi}}\left[e^{\frac{-(x-\mu)^2}{2\sigma^2}}\left(\frac{-(x-\mu)}{\sigma^2}\right)(x-\mu) + e^{\frac{-(x-\mu)^2}{2\sigma^2}}\right]$$

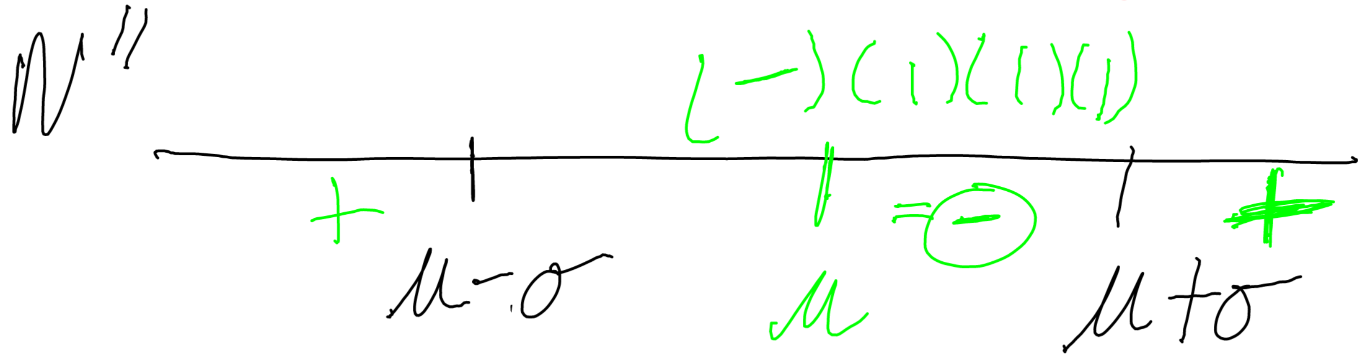
$$= \frac{-1}{\sigma^3\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}\left[-\frac{(x-\mu)^2}{\sigma^2} + 1\right]$$

$$= \cdots\cdots \left[1 - \left(\frac{x-\mu}{\sigma}\right)^2\right]$$

Now use $a^2 - b^2 = (a-b)(a+b)$

$$N''(x) = \underbrace{\frac{-1}{\sigma^3\sqrt{2\pi}}}_{-}\,\underbrace{e^{\frac{-(x-\mu)^2}{2\sigma^2}}}_{+}\underbrace{\left[1-\frac{x-\mu}{\sigma}\right]}_{=0 \text{ if}}\cdot\underbrace{\left[1+\frac{x-\mu}{\sigma}\right]}_{=0 \text{ if}}$$

$\frac{x-\mu}{\sigma} = 1$	$\frac{x-\mu}{\sigma} = -1$
$x-\mu = \sigma$	$x-\mu = -\sigma$
$x = \mu+\sigma$	$x = \mu-\sigma$



$\therefore$ N is concave up when $N''>0$

$\iff$ $x < \mu-\sigma$ or $x > \mu+\sigma$

N is concave down when

$$\mu-\sigma < x < \mu+\sigma$$

It means that there are points of inflection at $\mu \pm \sigma$