Some Mathematical Problems

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Example 1. *In the 12 days of Christmas song how many gifts does your true love have to buy in total? Consider using a loop within a loop! Bonus: Try to get your program to print the lyrics of the song.*

Example 2. *The Birthday Paradox*

`https://en.wikipedia.org/wiki/Birthday_problem`

is an unintuitive result in probability.

- Make a list of length n (think 50 people).
- Give each person a random number between 1 and 366 (think birthday).
- Have your program determine if there is a duplicate (two people with the same birthday). There may be a built in function to do this for you. You may want to use a sorting algorithm you can use. You may want to make the whole thing yourself.
- Once you have the above steps working put your whole program in a loop and run the experiment many times. This will give you an idea of the probability to have two people with the same birthday.
- With 23 people you should have a duplicate 50% of the time. With 50 people you should have a duplicate over 90% of the time.

Example 3. *Root Finding by Bisection.*

Suppose we want to find the solutions to

$$0 = x^3 - 3x^2 + 1.$$

Observe that this is the same as finding the zeros/roots of the graph

$$y = x^3 - 3x^2 + 1.$$

Notice also that when $x = 0$ we get the following y value

$$y = 0^3 - 3(0)^2 + 1 = 1$$

and for $x = 1$ we see

$$y = 1^3 - 3(1)^2 + 1 = -1.$$

So, the y value is positive at $x = 0$ and negative at $x = 1$. We suspect that the y value will be zero somewhere between $x = 0$ and $x = 1$. Let us investigate the y value when $x = 0.5$

$$y = (0.5)^3 - 3(0.5)^2 + 1 = 0.375$$

and so we have a positive y value at $x = 0.5$. Therefore, we expect the root to be between $x = 0.5$ and $x = 1$.

When we started we knew the root was between 0 and 1 but now we know it to be between 0.5 and 1. Write a program that performs this 'bisection' method a couple of times and narrows the range in which the zero lies. Check your answer with wolfram alpha.