Theme: Math Day 1 (Graph Theory)

Developed by: Matt Coles and Cole Zmurchok

Date: February 14, 2017

Location: Innovation Lab at Science World

Objectives

1. Appreciate mathematics as its own discipline with active research
2. Explain, using mathematical or logical reasoning, why you can't traverse the 7 bridges

Preparation for students

Think about any questions you may have for real live mathematicians. Examples include: Why did you choose mathematics? What do you do all day? Why can't I divide by zero?

Students should bring some *pencil crayons* (4 each will suffice [although don't tell them that]).

Timeline

4:30- 4:50 (20 mins): Hello and Introductions
4:50- 5:30 (40 mins): Four Colour Theorem
5:30- 5:50 (15 mins): Math Panel
5:50- 6:05 (15 mins): Break (informal conversations)
6:05- 6:45 (40 mins): Bridges of Königsberg

Homework

- Watch video for next week (see plan for Day 2)

Resources needed (separate into different activities)

- 4 colour theorem handouts (2x28) – Matt
- Bridges handout (28) – Matt
- Blank paper for colouring and bridges – Matt

Volunteer roles

Any Volunteers can feel free to wander, discuss, facilitate or participate!

Set up needed

No computers needed. Students arranged at tables for group work.

Math Day 1

Matt Coles and Cole Zmurchok

February 14, 2016

0 Things to Bring

- colour handouts (2×28) and blank paper (Matt)
- bridges handouts (28) - Matt
- a few pencil crayons (students)

1 Introductions (20 min)

- Everyone says hello. Grad students each say a little bit about their research area. Time for a few questions but larger question period will follow (we'll see how talkative they are).

2 The Four Colour Theorem (40 min)

- Give students handouts. First map of Europe and then abstract maps. Ask them to colour the maps with as few colours as possible.
- Have students in their groups draw their own map and then switch with another group. Students try to colour the maps with only 4 colours.
- Discussion on how one would need to prove such a theorem. Maybe a few comments on the idea of the proof
  - Conjecture 1852. 1880-1890 incorrect proofs. 1890 five colour theorem. 1960s, 1970s using computers. 1976 proof.

3 What being a Mathematician is like (20 min)

- Can vary in length depending on length of Introduction.
- Fairly open discussion. What do you think mathematicians do? What do you think mathematics is? Other questions.
- Students may come prepared with questions. Can think/pair/share to solicit new questions
- It would be nice to get different opinions from grad students

4 Break time! (10-15 min)

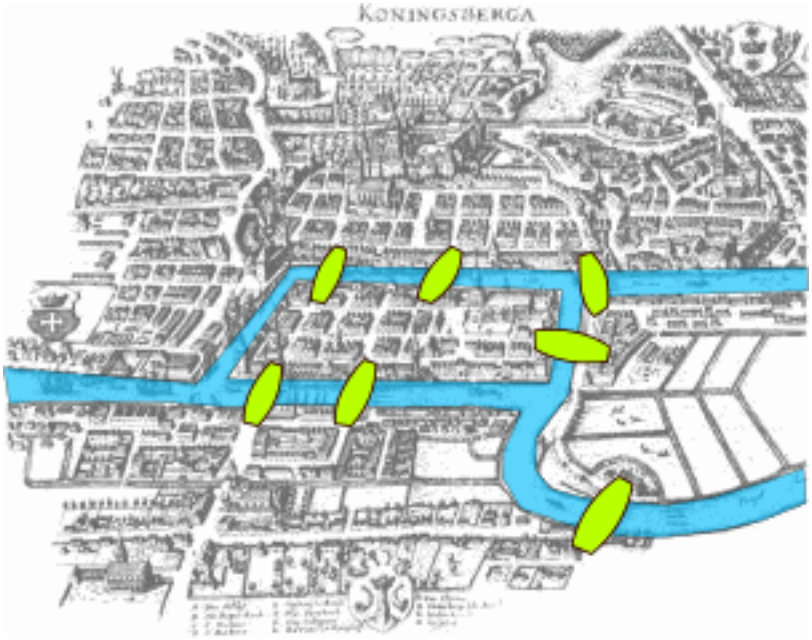
- Informal conversations.

5 The Bridges of Königsberg (40min)

- Draw the picture bellow and ask students to construct a walk that covers all bridges once and only once.
- After they become frustrated you can let it slip that such a walk is impossible. Transition into talking about why it is impossible. Or at least emphasize that proving it is impossible is something that needs to be done.
- Facilitate how you please but if you have trouble getting them going you can ask them to draw an additional bridge and then try to solve the problem. No matter what they draw I think it should work. Although they will need to choose the correct starting point. Ask why (after drawing a new bridge) starting in a place with an even number of bridges will not work.
- Can ask them to draw two bridges and insist that they start and finish in the same spot.
- See next page for figures.

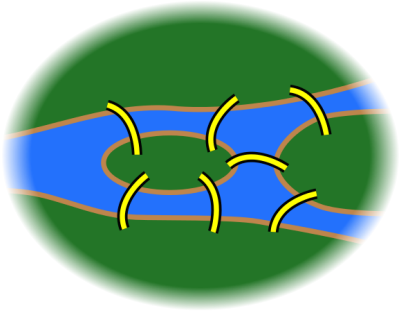
- The important thing to remember is that if you have a land mass with an odd number of bridges it has to be either the start or finish.

Observe the following map of Königsberg circa 1736:



Try to devise a walk where you travel over each of the 7 bridges once and only once. It doesn't matter where you start and where you end. You do not need to finish in the same place you started. So go and try it. Why are you still reading this? Come back once you've tried it.

You can also think about the city in the following simplified way:



Now that you've tried it you should realize several things. For starters, constructing such a walk is impossible. The question should really be: why is the above problem impossible? Or else: *prove* that no matter how you design a walk it will never cross all bridges exactly once.

Try adding another bridge. You should be able to solve the problem now but it matters where you start and where you finish.

Once you've solved it look at the places where you started and finished. How many bridges do those places have access to? Look at the intermediate land masses. How many bridges do they have access to?

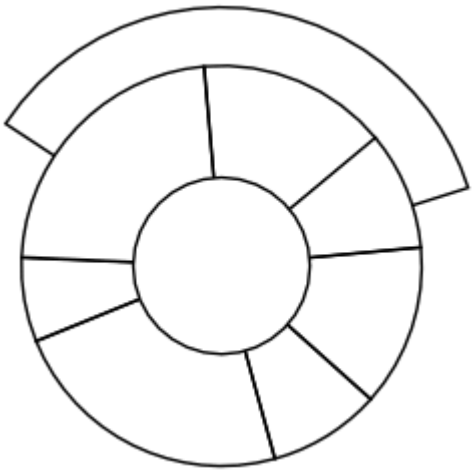
Try adding a second bridge so that you can start and finish at the same point. What do you notice now about the number of bridges?

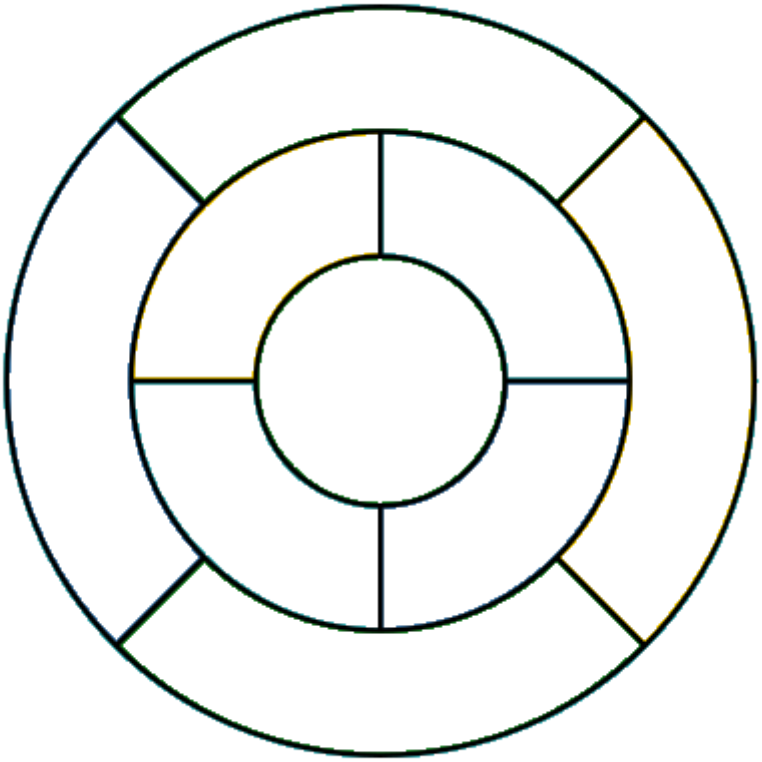
Check out the Wikipedia page:

`https://en.wikipedia.org/wiki/Seven_Bridges_of_K%C3%B6nigsberg`

for a full explanation as well as additional problems.


Poland
Germany
Czech
Republic
Slovakia
Austria
Hungary
Switzer
land
Slovenia
Italy





The Bridges of Königsberg

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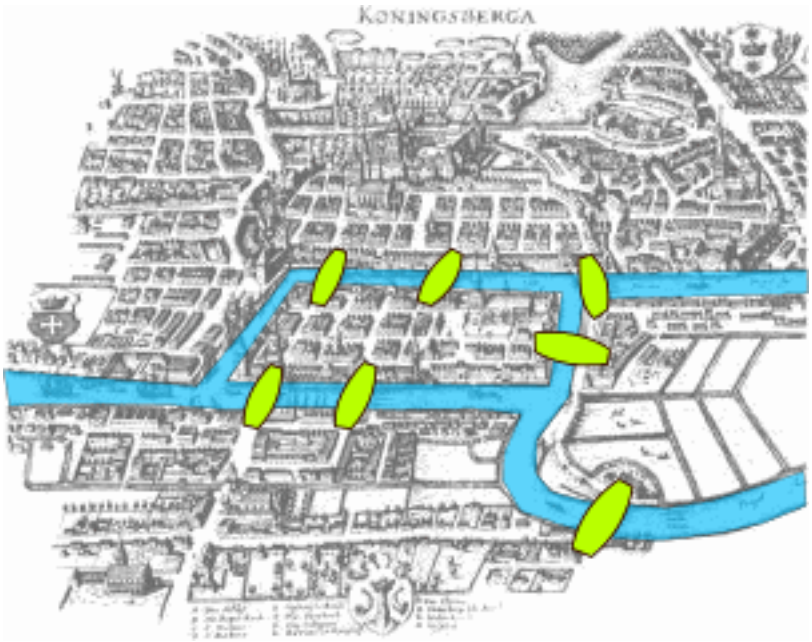

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