Theme: Iteration
Developed by: Matt Coles and Cole Zmurchok
Date: January 21, 2017
Location: Innovation Lab at Science World or other destination
Objectives
1. By the end of the session students will be able to explain what an iterative method is and solve simple math problems using one 2. By the end of the session students will be able to calculate the change in a quantity from day to day given a rule and using a spreadsheet.
Preparation for students
Watch video about the bisection method on Matt's website: www.math.ubc.ca/~colesmp/FSL
Timeline
4:30-5:30 (60 mins): Introduction to iteration activity. Students work to solve iteration problems on handout, including group discussion and summary. 6:00-7:00 (60 mins): Introduction to Juliet and Romeo activity. Students complete introductory activity (how a rumor spreads) on handout with quick group summary. Juliet and Romeo's relationship introduction with spreadsheet demonstration. Students work on exploring what happens to their relationship over time.
Homework
Possible: write a blog post reflecting on what you learned in one of the four math activities (Four Colour Theorem, Bridges of Königsberg, Iteration, Juliet and Romeo).
Resources needed (separate into different activities)
● We will use the science world computers. Students are encouraged to bring their own. We'll want at least one computer per three students ● We will bring handouts for both activities (Iteration and Juliet and Romeo)
Volunteer roles
Feel free to participate or circulate and help with spreadsheet tips!
Set up needed
Just the computers and students working in small groups. Projector to show spreadsheet demonstration for Juliet and Romeo activity.

Math Day 2

Matt Coles and Cole Zmurchok

February 21, 2017

0 Things to bring

- Handouts for iteration activity
- Handouts for Romeo and Juliet
- Spreadsheet capable device (students), i.e., laptop or Science World computers.

1 Iteration Activity (60 min)

- (5 min) "You don't have to be perfect, you just have to be getting better."

  Learning Objective: Describe the phrase 'iterative process.' Identify connections between the following activities.

  Students are told not to get distracted by details and try to look instead for the big picture.

- (15 min) Activity: Students (with their facilitator) think about solving the following two problems. After they figure out the questions we can start to think about the whole process (algorithm) as an object itself.

  1. Find 240/7 (as a decimal, no calculator).
     Is it between: 10 and 20, 20 and 30, 30 and 40, or 40 and 50?
     Is it between: 33 and 34, 34 and 35, 35 and 36, or 36 and 37?

Is it between: 34.0 and 34.1, 34.1 and 34.2, 34.2 and 34.3, or 34.3 and 34.4?

Instead of having them do long division have them guess and check to hint at the bisection method.

2. Find $\sqrt{7}$ (as a decimal, calculator just for $*, /, +, -$).

   Find a number smaller than $\sqrt{7}$ and a number bigger than $\sqrt{7}$.

   Make this range containing $\sqrt{7}$ smaller.

   Describe the process you are using to find square roots by hand.

- (15 min) Next the harder activity:

  1. Find where the graph of $y = x^3 - 3x^2 + 1$ crosses the x-axis (calculator encouraged).

     Can you get a rough idea of what the graph looks like (plot some points).

     Can you find where the graph is positive? Negative?

     Can you find a range of x-values in which a zero must lie. Can you make the range smaller?

- (15 min) Debrief. Have students think by themselves about how to describe an iterative process. Then have a group discussion and list the ideas from the group. Look for words like: guess (initial guess), bounds, trial and error, bisection, improvement, convergence, 'get's better', steps...

  Finally ask who might use an iterative method? Ask students to come up with examples of iteration from both science and day-to-day life. Ex. Evolution, calculating π.

- (10 min) As a group or in small groups we can sketch out pseudo-code to perform the bisection method to find the roots of our cubic function.

2 Juliet and Romeo (60 min)

Learning objective: Be able to calculate the change in a quantity from day to day given a rule by and by using a spreadsheet.

- (25 min) You find out a secret on Monday. On Tuesday, you tell two of your friends. On Wednesday, everyone who knows the secret tells two more people. How many people know the secret on Thursday? Friday? Saturday?

  FSL Fellow: Generalize from data to the rule $x_{n+1} = 3x_n$ via discussion on white board.

  What if you tell a different number of people each day? What if more than one person knows the rumor on Monday? Introduce spreadsheets as a quick way to calculate $x_{n+1} = rx_n$ for any r and x_0?

- (5 min) Juliet and Romeo's relationship. Explain that Juliet and Romeo can have positive or negative feelings for each other, and they may change each day.

  For example:

$$\begin{aligned} J_{n+1} &= \frac{1}{2} R_n \\ R_{n+1} &= 2J_n \end{aligned}$$

  Faciliate a discussion about the meaning of this *mathematical model*.

  - Juliet and Romeo's feelings for each other depend only how the other felt for them yesterday.
  - Romeo over-reacts to Juliet's feelings ($2J_n$), but Juliet doesn't let Romeo get under her skin too much ($\frac{1}{2}R_n$).

- (25 min) Use a spreadsheet to determine what happens to Juliet and Romeo's relationship as the days go by. Suppose that Juliet initially dislikes Romeo ($J_0 = -1$) and that Romeo initially likes Juliet ($R_0 = 1$). Example Spreadsheet.

  Follow up questions:

  1. Show possible ways to visualize data, for example, using by highlighting the sequence and clicking on Line Graph.
  2. What if Romeo's feelings respond to his own feelings and how Juliet feels for him? How does that change what happens to their relationship over time?

3. What if Juliet's feelings also depend on her own feelings and how Romeo feels for her? What happens over time?

   For example,

$$R_{n+1} = 0.5R_n + 0.7J_n,$$
$$J_{n+1} = 0.7R_n + 0.9J_n.$$

   What happens if the *parameters*, 0.5, 0.7, 0.7 and 0.9, change?

4. Describe what happens to Juliet and Romeo's relationship,

$$R_{n+1} = aR_n + bJ_n,$$
$$J_{n+1} = cR_n + dJ_n,$$

   if a, b, c, and d can change. Try

   – $a = 0.5$, $b = 0.2$, $c = 0.5$, $d = 0.7$
   – $a = 1$, $b = 0.2$, $c = -0.2$, $d = 1$
   – $a = 0.8$, $b = -1.1$, $c = 1$, $d = -0.2$

- (5 min) Wrap-up. What did you find? Anything cool? Thoughts on application to study? Examples include: drug levels in the body, population dynamics, predator-prey dynamics, the evolution of a gene, the spread of disease.

1. Consider the number $240/7$. Let's try to figure out some information about this number without using long division.

   (a) Demonstrate (with some multiplication) that $240/7$ is bigger than 30.

   (b) Demonstrate that $240/7$ is smaller than 40.

   (c) Show that $34 < 240/7 < 35$.

2. Let us now consider the number $\sqrt{7}$. We will try to compute without using a calculator.

   (a) Find a number smaller than $\sqrt{7}$. Show how you know.

   (b) Find a number larger than $\sqrt{7}$. Show how you know.

   (c) Now that you have a range in which $\sqrt{7}$ lies, try to make this range smaller.

   (d) Explain how you can continue this process to make your range containing $\sqrt{7}$ as small as you like. This method can be called the Bisection Method.

3. Now for the hard problem. Imagine we wanted to find the x-intercepts (zeros, roots) of the equation

$$y = x^3 - 3x^2 + 1.$$

That is, we want to solve

$$x^3 - 3x^2 + 1 = 0.$$

Feel free to plot a few (x, y) coordinates to get a sense of what this graph looks like. Feel free to use a calculator (but no graphing calculator).

   (a) Find a point x whose corresponding y coordinate is positive.

   (b) Find a point x whose corresponding y coordinate is negative.

   (c) Now that your root is contained in an interval, improve your approximation using the Bisection Method.

   (d) There are 3 roots total. Try to approximate them all. Check your answers with wolfram alpha. If you're into programming try to sketch out some pseudo-code to perform the algorithm. If you're not into programming write some English sentences to teach your friend how to perform the method.

1. You find out a secret on Monday. On Tuesday, you tell two of your friends. On Wednesday, everyone who knows the secret tells two more people. How many people know the secret on Thursday? Friday? Saturday?

   Fill out the following table:

Day of the week	Day number	Number of people that know the secret
Monday	1	1
Tuesday	2	3
Wednesday		

   If x_n is the number of people who know the secret on day number n, how many people know the secret on the next day, day number $n+1$?

   $$x_{n+1} =$$

2. Juliet and Romeo have can have positive or negative feelings for each other. At the beginning, Juliet dislikes Romeo, $J_0 = -1$, and Romeo likes Juliet, $R_0 = 1$.

   Suppose that Juliet and Romeo's feelings are governed by the following relationships:

   $$J_{n+1} = \frac{1}{2}R_n$$
   $$R_{n+1} = 2J_n$$

   Explain what both equations in this *mathematical model* mean.

3. Use a spreadsheet to determine what happens over 30 days to their feelings using the following rules:

$$J_{n+1} = \frac{1}{2}R_n$$
$$R_{n+1} = 2J_n$$

Explain what happens to their feelings for each other. Try to make a line graph of the results.

4. What if Juliet's feelings depend not only on her own feelings but also how Romeo feels for her? For example, suppose that their feelings are described by the following rules:

$$J_{n+1} = \frac{1}{2}R_n - 2J_n$$
$$R_{n+1} = 2J_n$$

Explain what happens to their feelings over time. Try to make a line graph of the results.

5. Suppose that

$$\begin{aligned} R_{n+1} &= aR_n + bJ_n, \\ J_{n+1} &= cR_n + dJ_n, \end{aligned}$$

where a, b, c, and d can change. Try

(a) $a = 0.5$, $b = 0.7$, $c = 0.7$, $d = 0.9$

(b) $a = 0.5$, $b = 0.2$, $c = 0.5$, $d = 0.7$

(c) $a = 1$, $b = 0.2$, $c = -0.2$, $d = 1$

(d) $a = 0.8$, $b = -1.1$, $c = 1$, $d = -0.2$

and describe what happens to their feelings for each other in each case.