Iteration and the Bisection Method

1. Consider the number 240/7. Let's try to figure out some information about this number without using long division.

   (a) Demonstrate (with some multiplication) that 240/7 is bigger than 30.

   (b) Demonstrate that 240/7 is smaller than 40.

   (c) Show that $34 < 240/7 < 35$.

2. Let us now consider the number $\sqrt{7}$. We will try to compute without using a calculator.

   (a) Find a number smaller than $\sqrt{7}$. Show how you know.

   (b) Find a number larger than $\sqrt{7}$. Show how you know.

   (c) Now that you have a range in which $\sqrt{7}$ lies, try to make this range smaller.

   (d) Explain how you can continue this process to make your range containing $\sqrt{7}$ as small as you like. This method can be called the Bisection Method.

3. Now for the hard problem. Imagine we wanted to find the x-intercepts (zeros, roots) of the equation

$$y = x^3 - 3x^2 + 1.$$

That is, we want to solve

$$x^3 - 3x^2 + 1 = 0.$$

Feel free to plot a few (x, y) coordinates to get a sense of what this graph looks like. Feel free to use a calculator (but no graphing calculator).

(a) Find a point x whose corresponding y coordinate is positive.

(b) Find a point x whose corresponding y coordinate is negative.

(c) Now that your root is contained in an interval, improve your approximation using the Bisection Method.

(d) There are 3 roots total. Try to approximate them all. Check your answers with wolfram alpha. If you're into programming try to sketch out some pseudo-code to perform the algorithm. If you're not into programming write some English sentences to teach your friend how to perform the method.