Homeworks:

1. Put your name and student number on your solutions.

2. **Staple** your homework!

3. Last week's homework has been graded and is available at the MLC (maybe tomorrow).

4. Solutions to the graded problems (5 & 6) are available online.

5. For problem 5 (intersection): the answer must be a function in $t$ (so no $\pm \sqrt{\cdots}$)!

→ Draw picture
Let's think a bit about acceleration:

\[ \ddot{a} = \dddot{v} = \dddot{r} \]

Driving a car, we are familiar with two types of acceleration:

- Pressing on the gas pedal, we are pushed back into the seat.

  acceleration in the \( \hat{r} \) direction
acceleration in the T direction

Driving (at constant speed) around a corner, we are pushed to the side.

normal force

acceleration in the N direction

More generally, we experience acceleration that is a combination of these.

Let's derive a formula:

\[ \vec{r}' = \vec{v}, \quad |\vec{r}'| = v - \frac{d\vec{s}}{dt}, \quad \vec{T} = \frac{\vec{r}'}{|\vec{r}'|} \]

\[ \vec{a} = \vec{v}' = \frac{d\vec{v}}{dt} \cdot \vec{T} + v \cdot \vec{T}' \]

\[ \vec{v} = v \cdot \frac{\vec{r}}{|\vec{r}|} \]
\[ \frac{d^2 s}{dt^2} \cdot \frac{\vec{T}}{1} + \nu \cdot |\vec{T}'| \cdot \vec{N} \]

\[ \vec{N} = \frac{\vec{T}'}{|\vec{T}'|} \]

\[ \kappa = \frac{|\vec{T}'|}{\nu} \]

\[ a_T \]

\[ a_N \]

tangential component
derivative of speed

normal component
the sharper the turn and the higher the speed, the more acceleration

Example: Find the tangential and normal components of the acceleration of a car driving up a spiral parking ramp.
\[ \vec{r}(t) = \langle \cos(t), \sin(t), t \rangle \]

Find the tangential and normal components of the acceleration.

\[ \vec{r}'(t) = \langle -\sin(t), \cos(t), 1 \rangle \]

\[ v = |\vec{r}'| = \sqrt{\sin^2 t + \cos^2 t + 1} = \sqrt{2}, \quad \frac{dv}{dt} = 0 \]

no tangential acceleration,

the car goes at constant speed

\[ \vec{T} = \frac{1}{12} \langle -\sin t, \cos t, 1 \rangle \]

\[ \vec{T}' = \frac{1}{12} \langle -\cos t, \sin t, 0 \rangle \quad |\vec{T}'| = \frac{1}{12} \sqrt{\cos^2 t + \sin^2 t} = \frac{1}{12} \]

\[ k = \frac{|\vec{T}'|}{v} = \frac{\frac{1}{12}}{\sqrt{2}} = \frac{1}{12} \sqrt{2} \quad i \quad m^2 \quad 1 \]
\[ \vec{a} = k \cdot \vec{v} = \frac{1}{2} \cdot \sqrt{2}^2 = 1 \]