Very short answer questions

1. 2 marks Each part is worth 1 mark. Please write your answers in the boxes. **Marking scheme:** 1 for each correct, 0 otherwise

(a) Find the domain of continuity for the function \( f(x) = \log(4x^2 - 1) \).

Answer: \((-\infty, -\frac{1}{2}) \cup (\frac{1}{2}, +\infty)\)

**Solution:** The function is continuous when \( 4x^2 - 1 > 0 \), i.e. \( (2x - 1)(2x + 1) > 0 \), which yields the intervals \((-\infty, -\frac{1}{2}) \cup (\frac{1}{2}, +\infty)\).

(b) Compute \( \lim_{t \to 1} \sqrt{5x^3 + 4} \).

Answer: 3

**Solution:**

\[
\lim_{t \to 1} \sqrt{5x^3 + 4} = \sqrt{\lim_{t \to 1} (5x^3 + 4)} = \sqrt{5 \lim_{t \to 1} (x^3) + 4} = \sqrt{9} = 3.
\]

Short answer questions — you must show your work

2. 4 marks Each part is worth 2 marks.

(a) Compute the limit \( \lim_{x \to -3} \frac{x^2 - 9}{x + 3} \).

**Solution:** If try naively then we get 0/0, so we simplify first:

\[
\frac{x^2 - 9}{x + 3} = \frac{(x - 3)(x + 3)}{(x + 3)} = x - 3
\]

Hence the limit is \( \lim_{x \to -3} (x - 3) = -6 \). **Marking scheme:** 1 for factoring+cancelling, 1 for answer. If answer with no working then 0.

(b) Find all values of \( c \) such that the following function is continuous:

\[
f(x) = \begin{cases} 
  x^2 + c & \text{if } x < c \\
  2cx - 2 & \text{if } x \geq c
\end{cases}
\]
Solution: The function is continuous for \( x \neq c \) since each of those two branches are polynomials. So, the only question is whether the function is continuous at \( x = c \); for this we need
\[
\lim_{x \to c^-} f(x) = f(c) = \lim_{x \to c^+} f(x).
\]
We compute
\[
\lim_{x \to c^-} f(x) = \lim_{x \to c^-} x^2 + c = c^2 + c;
\]
\[
f(c) = 2c \cdot c - 2 = 2c^2 - 2;
\]
\[
\lim_{x \to c^+} f(x) = \lim_{x \to c^+} 2cx - 2 = 2c^2 - 2.
\]
So, we need \( c^2 + c = 2c^2 - 2 \), which yields \( c^2 - c - 2 = (c - 2)(c + 1) = 0 \), i.e. \( c = -1 \) or \( c = 2 \).

Marking scheme:

- 1 mark for writing the condition for continuity \( \lim_{x \to c^-} f(x) = f(c) = \lim_{x \to c^+} f(x) \).
- 1 mark for solving correctly and finding both solutions \( c = -1 \) and \( c = 2 \).

Long answer question — you must show your work

3. **4 marks** Compute the limit \( \lim_{x \to 1} \frac{\sqrt{x+2} - \sqrt{4-x}}{x-1} \).

Solution: If we try to do the limit naively we get 0/0. Hence we must simplify. **Marking scheme**: If multiply by correct conjugate then 2 marks. If multiply by something close to correct then 1. Else 0.

\[
\frac{\sqrt{x+2} - \sqrt{4-x}}{x-1} = \frac{\sqrt{x+2} - \sqrt{4-x}}{x-1} \cdot \frac{\sqrt{x+2} + \sqrt{4-x}}{\sqrt{x+2} + \sqrt{4-x}}
\]
\[
= \frac{(x+2) - (4-x)}{(x-1)(\sqrt{x+2} + \sqrt{4-x})}
\]
\[
= \frac{2x - 2}{(x-1)(\sqrt{x+2} + \sqrt{4-x})}
\]
\[
= \frac{2}{\sqrt{x+2} + \sqrt{4-x}}
\]

Marking scheme: If correct simplification then 1 mark, else 0. So the limit is
\[
\lim_{x \to 1} \frac{\sqrt{x+2} - \sqrt{4-x}}{x-1} = \lim_{x \to 1} \frac{2}{\sqrt{x+2} + \sqrt{4-x}}
\]
\[
= \frac{2}{\sqrt{3} + \sqrt{3}}
\]
\[
= \frac{1}{\sqrt{3}}
\]

Marking scheme: 1 for answer.