Very short answer questions

1. [2 marks] Each part is worth 1 mark. Please write your answers in the boxes. **Marking scheme:** 1 for each correct, 0 otherwise
   
   (a) Evaluate \( \tan \left( \frac{\pi}{3} \right) \).
   
   **Solution:**
   
   \[
   \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2} \quad \cos \frac{\pi}{3} = \frac{1}{2} \quad \text{so } \tan \frac{\pi}{3} = \sqrt{3}
   \]
   
   Else draw the appropriate 2 : 1 : \( \sqrt{3} \) triangle.

   (b) Compute \( \lim_{t \to -1} \left( \frac{t - 2}{t + 3} \right) \).

   **Solution:**
   
   \[
   \lim_{t \to -1} \left( \frac{t - 2}{t + 3} \right) = \frac{\lim_{t \to -1} (t - 2)}{\lim_{t \to -1} (t + 3)} = \frac{-3}{2}.
   \]

Short answer questions — you must show your work

2. [4 marks] Each part is worth 2 marks.

   (a) Find all solutions to \( x^3 - 3x^2 - x + 3 = 0 \)

   **Solution:**
   
   \[
   x^3 - 3x^2 - x + 3 = x^2(x - 3) - (x - 3) = (x^2 - 1)(x - 3) = (x - 1)(x + 1)(x - 3)
   \]
   
   So solutions are \( x = -1, 1, 3 \). **Marking scheme:** If all 3 soln then 2 marks. If some factoring and 1 or 2 solutions, then 1 mark. If 1 or 2 solution and no working, then 0. If no solutions then 0. ALSO — if a student factors correctly but then gets signs wrong on roots or does not give roots, then give 1 mark.

   (b) Compute the limit \( \lim_{x \to 2} \frac{x - 2}{x^2 - 4} \). 

   **Solution:** 
   
   \[
   \lim_{x \to 2} \frac{x - 2}{x^2 - 4} = \lim_{x \to 2} \frac{(x - 2)}{(x - 2)(x + 2)} = \lim_{x \to 2} \frac{1}{x + 2} = \frac{1}{4}
   \]
Solution: If try naively then we get $0/0$, so we simplify first:
\[
\frac{x - 2}{x^2 - 4} = \frac{x - 2}{(x - 2)(x + 2)} = \frac{1}{x + 2}
\]
Hence the limit is $\lim_{x \to 2} \frac{1}{x + 2} = 1/4$. **Marking scheme:** 1 for factoring+cancelling, 1 for answer. If answer with no working then 0.

Long answer question — you must show your work

3. **4 marks** Compute the limit $\lim_{x \to 1} \frac{\sqrt{x + 2} - \sqrt{4 - x}}{x - 1}$.

**Solution:** If we try to do the limit naively we get $0/0$. Hence we must simplify. **Marking scheme:** If multiply by correct conjugate then 2 marks. If multiply by something close to correct then 1. Else 0.
\[
\frac{\sqrt{x + 2} - \sqrt{4 - x}}{x - 1} = \frac{\sqrt{x + 2} - \sqrt{4 - x}}{x - 1} \cdot \frac{\sqrt{x + 2} + \sqrt{4 - x}}{\sqrt{x + 2} + \sqrt{4 - x}}
\]
\[
= \frac{2 - 2}{(x - 1)(\sqrt{x + 2} + \sqrt{4 - x})}
\]
\[
= \frac{2}{x - 1}(\sqrt{x + 2} + \sqrt{4 - x})
\]
\[
= 2
\]
\[
\frac{\sqrt{x + 2} + \sqrt{4 - x}}{\sqrt{3} + \sqrt{3}}
\]
\[
= \frac{1}{\sqrt{3}}
\]
**Marking scheme:** 1 for answer.
Very short answer questions

1. [2 marks] Each part is worth 1 marks. Please write your answers in the boxes. **Marking scheme:** 1 for each correct, 0 otherwise
   
   (a) Compute \( \tan \left( \frac{\pi}{6} \right) \).

   **Answer:** \( \frac{1}{\sqrt{3}} \)

   **Solution:**
   
   \[
   \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2} \quad \sin \frac{\pi}{6} = \frac{1}{2} \quad \text{so} \quad \tan \frac{\pi}{3} = \frac{1}{\sqrt{3}}
   \]

   Else draw the appropriate 2 : 1 : \( \sqrt{3} \) triangle.

   (b) Compute \( \lim_{t \to -2} \left( \frac{t - 5}{t + 4} \right) \).

   **Answer:** \(-7/2\)

   **Solution:**
   
   \[
   \lim_{t \to -2} \left( \frac{t - 5}{t + 4} \right) = \frac{\lim_{t \to -2} (t - 5)}{\lim_{t \to -2} (t + 4)} = -7/2.
   \]

Short answer questions — you must show your work

2. [4 marks] Each part is worth 2 marks.
   
   (a) Find all solutions to \( x^3 - x^2 - 4x + 4 = 0 \)

   **Solution:**
   
   \[
   x^3 - x^2 - 4x + 4 = x^2(x - 1) - 4(x - 1) = (x^2 - 4)(x - 1) = (x - 2)(x + 2)(x - 1)
   \]

   So solutions are \( x = 2, -2, 1 \). **Marking scheme:** If all 3 soln then 2 marks. If some factoring and 1 or 2 solutions, then 1 mark. If 1 or 2 solution and no working, then 0. If no solutions then 0.

   (b) Compute the limit \( \lim_{x \to 3} \frac{x - 3}{x^2 - 9} \)

   **Solution:** If try naively then we get 0/0, so we simplify first:
   
   \[
   \frac{x - 3}{x^2 - 9} = \frac{x - 3}{(x - 3)(x + 3)} = \frac{1}{x + 3}
   \]
Hence the limit is \( \lim_{x \to 3} \frac{1}{x + 3} = 1/6 \). **Marking scheme:** 1 for factoring+cancelling, 1 for answer. If answer with no working then 0.

Long answer question — you must show your work

3. **4 marks** Compute the limit \( \lim_{x \to 3} \frac{\sqrt{x - 2} - \sqrt{4 - x}}{x - 3} \).

**Solution:** If we try to do the limit naively we get \( 0/0 \). Hence we must simplify. **Marking scheme:** If multiply by correct conjugate then 2 marks. If multiply by something close to correct then 1. Else 0.

\[
\frac{\sqrt{x - 2} - \sqrt{4 - x}}{x - 3} = \frac{\sqrt{x - 2} - \sqrt{4 - x}}{x - 3} \cdot \frac{\sqrt{x - 2} + \sqrt{4 - x}}{\sqrt{x - 2} + \sqrt{4 - x}}
\]

\[
= \frac{(x - 2) - (4 - x)}{(x - 3)(\sqrt{x - 2} + \sqrt{4 - x})}
\]

\[
= \frac{2x - 6}{(x - 3)(\sqrt{x - 2} + \sqrt{4 - x})}
\]

\[
= \frac{2}{\sqrt{x - 2} + \sqrt{4 - x}}
\]

**Marking scheme:** If correct simplification then 1 mark, else 0. So the limit is

\[
\lim_{x \to 3} \frac{\sqrt{x - 2} - \sqrt{4 - x}}{x - 3} = \lim_{x \to 3} \frac{2}{\sqrt{x - 2} + \sqrt{4 - x}}
\]

\[
= \frac{2}{1 + 1}
\]

\[
= 1.
\]

**Marking scheme:** 1 for answer.
Very short answer questions

1. 2 marks Each part is worth 1 mark. Please write your answers in the boxes. **Marking scheme:** 1 for each correct, 0 otherwise
   
   (a) Evaluate $\csc \left( \frac{\pi}{3} \right)$.

   **Solution:**
   
   $$\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2} \quad \csc \theta = \frac{1}{\sin \theta}$$
   
   So $\csc \frac{\pi}{3} = \frac{2}{\sqrt{3}}$
   
   Else draw the appropriate $2 : 1 : \sqrt{3}$ triangle.

   **Answer:** $\frac{2}{\sqrt{3}}$

   (b) Compute $\lim_{t \to -1} \left( \frac{t^2}{t - 1} \right)$.

   **Solution:**
   
   $$\lim_{t \to -1} \left( \frac{t^2}{t - 1} \right) = \frac{\lim_{t \to -1} (t^2)}{\lim_{t \to -1} (t - 1)} = -1/2.$$

   **Answer:** $-1/2$

Short answer questions — you must show your work

2. 4 marks Each part is worth 2 marks.
   
   (a) Let $f(x) = 3x^2 - 7x - 3$ and $g(x) = 2x^2 - 6x + 3$. Find all values of $x$ for which $f(x) = g(x)$.

   **Solution:**
   
   $$3x^2 - 7x - 3 = 2x^2 - 6x + 3 \quad \Leftrightarrow x^2 - x - 6 = 0 \quad \Leftrightarrow (x - 3)(x + 2) = 0$$
   
   So solutions are $x = 3, -2$. **Marking scheme:** If both soln then 2 marks. If only one soln: 1 mark with work, 0 marks with no work. If correct work but wrong signs on solutions, 1 mark. If set equal to each other and tried to solve but couldn’t, 1 mark. A carefully drawn graph counts as work, but a sloppy or careless graph does not.

   (b) Compute the limit $\lim_{x \to -2} \frac{x + 2}{x^2 - 4}$
Solution: If try naively then we get $0/0$, so we simplify first:

\[
\frac{x + 2}{x^2 - 4} = \frac{x + 2}{(x - 2)(x + 2)} = \frac{1}{x - 2}
\]

Hence the limit is \(\lim_{x\to-2} \frac{1}{x - 2} = -1/4\). **Marking scheme:** 1 for factoring+cancelling, 1 for answer. If answer with no working then 0. Correct except for sign, 1 mark.

Long answer question — you must show your work

3. 4 marks Compute the limit \(\lim_{x\to1} \frac{\sqrt{3x + 5} - \sqrt{2x + 6}}{x - 1}\).

Solution: If we try to do the limit naively we get $0/0$. Hence we must simplify. **Marking scheme:** If multiply by correct conjugate then 2 marks. If multiply by something close to correct then 1. Else 0.

\[
\frac{\sqrt{3x + 5} - \sqrt{2x + 6}}{x - 1} = \frac{\sqrt{3x + 5} - \sqrt{2x + 6}}{x - 1} \cdot \frac{\sqrt{3x + 5} + \sqrt{2x + 6}}{\sqrt{3x + 5} + \sqrt{2x + 6}}
\]

\[
= \frac{(3x + 5) - (2x + 6)}{(x - 1)(\sqrt{3x + 5} + \sqrt{2x + 6})}
\]

\[
= \frac{x - 1}{(x - 1)(\sqrt{3x + 5} + \sqrt{2x + 6})}
\]

\[
= \frac{1}{\sqrt{3x + 5} + \sqrt{2x + 6}}
\]

**Marking scheme:** If correct simplification then 1 mark, else 0. So the limit is

\[
\lim_{x\to1} \frac{\sqrt{3x + 5} - \sqrt{2x + 6}}{x - 1} = \lim_{x\to1} \frac{1}{\sqrt{3x + 5} + \sqrt{2x + 6}} = \frac{1}{\sqrt{8} + \sqrt{8}} = \frac{1}{2\sqrt{8}} = \frac{1}{4\sqrt{2}}
\]

**Marking scheme:** 1 for answer. Simplification not necessary.
Very short answer questions

1. (2 marks) Each part is worth 1 mark. Please write your answers in the boxes. **Marking scheme:** 1 for each correct, 0 otherwise

   (a) Evaluate \( \tan \left( \frac{3\pi}{4} \right) \).

   **Answer:** \(-1\)

   **Solution:**
   \[
   \sin\left(\frac{3\pi}{4}\right) = \frac{1}{\sqrt{2}} \quad \cos\left(\frac{3\pi}{4}\right) = -\frac{1}{\sqrt{2}}
   \]
   so \( \tan\left(\frac{3\pi}{4}\right) = -1 \)

   (b) Compute \( \lim_{t \to 2} \sqrt{2t^3 - 16} \).

   **Answer:** 0

   **Solution:**
   \[
   \lim_{t \to 2} \sqrt{2t^3 - 16} = \sqrt{\lim_{t \to 2} 2t^3 - 16} = \sqrt{16 - 16} = 0
   \]

Short answer questions — you must show your work

2. (4 marks) Each part is worth 2 marks.

   (a) Find all \( x \) such that \( x^2 + 5x + 6 > 0 \).

   **Solution:**
   \[
   x^2 + 5x + 6 = (x + 2)(x + 3)
   \]
   So the expression is positive for \( x < -3 \) or \( x > -2 \). **Marking scheme:** Give 1 mark for factoring. Give the second mark for correct intervals (written as inequalities, intervals or marked on the number line). Give 1 mark total if factoring is incorrect, but the intervals are correct for their factoring. Give 0 for answer without any work.

   (b) Compute the limit \( \lim_{x \to -7} \frac{2x + 14}{x^2 - 49} \).

   **Solution:** If try naively then we get 0/0, so we simplify first:
   \[
   \frac{2x + 14}{x^2 - 49} = \frac{2(x + 7)}{(x + 7)(x - 7)} = \frac{2}{x - 7}
   \]
Hence the limit is \( \lim_{x \to -7} \frac{2}{x - 7} = -\frac{2}{14} = -\frac{1}{7} \). **Marking scheme:** 1 for factoring+cancelling, 1 for answer. If answer with no working then 0.

**Long Answer Question — You Must Show Your Work**

3. **4 marks** Compute the limit \( \lim_{x \to -1} \frac{x + 1}{\sqrt{x^2 + 15} - 4} \).

**Solution:** If we try to do the limit naively we get 0/0. Hence we must simplify. **Marking scheme:** If multiply by correct conjugate then 2 marks. If multiply by something close to correct then 1. Else 0.

\[
\frac{x + 1}{\sqrt{x^2 + 15} - 4} = \frac{x + 1}{\sqrt{x^2 + 15} - 4} \cdot \frac{\sqrt{x^2 + 15} + 4}{\sqrt{x^2 + 15} + 4} \\
= \frac{(x + 1)(\sqrt{x^2 + 15} + 4)}{(x^2 + 15) - 4^2} \\
= \frac{(x + 1)(\sqrt{x^2 + 15} + 4)}{(x^2 - 1)} \\
= \frac{(x + 1)(\sqrt{x^2 + 15} + 4)}{(x + 1)(x - 1)} \\
= \frac{\sqrt{x^2 + 15} + 4}{x - 1}
\]

**Marking scheme:** If correct simplification then 1 mark, else 0. So the limit is

\[
\lim_{x \to -1} \frac{x + 1}{\sqrt{x^2 + 15} - 4} = \lim_{x \to -1} \frac{\sqrt{x^2 + 15} + 4}{x - 1} \\
= \frac{8}{-2} \\
= -4
\]

**Marking scheme:** 1 for answer.