Lecture 1

(3 dim coordinates system)

R^3

Points in space

P = (a, b, c)

3 mutually perp. directed line segments at origin:
the standard coordinate axes (a right hand frame)
Start at origin, then

\[
\begin{cases}
\text{go } a \text{ units in } x \text{ direction, then} \\
\quad \text{b units in } y \\
\quad \text{c units in } z
\end{cases}
\]

eg) \(Q(1,5,3), \quad P(1,-5,2)\)
note:  - Q sits 3 units above pt.
      (1,5) in x y plane.
- could also have moved first
  in y direction, then ---
  or first
  in z direction, then ---
  to plot P and Q.

* distance formula:

distance between \( P(a_1, b_1, c_1) \) \& \( Q(a_2, b_2, c_2) \)

is \[
\text{dist}(P,Q) = \sqrt{(a_1-a_2)^2 + (b_1-b_2)^2 + (c_1-c_2)^2}
\]
Proof:

Get from P to Q by moving in x then y then z direction.

\[ E^2 = D^2 + C^2 \checkmark \]
\[ = D^2 + (A^2 + B^2) \]
\[ = (c_1 - c_2)^2 + (a_1 - a_2)^2 + (b_1 - b_2)^2 \]
An equation \( F(x, y, z) = 0 \) in \( x, y, z \), defines a surface \( S \) in space; namely, \( S \) is set of all points \((x, y, z)\) whose coords satisfy equation!

eg) Sketch the following surfaces in space

a) \( z = 1 \)

b) \( \frac{x^2}{4} + y^2 = 1 \)

c) \( (x-2)^2 + (y-2)^2 + (z-3)^2 = 1 \)

d) \( z = x^2 + y^2 \)
d) Here we must use the technique of "tracing". Reduces problem to sketching curves in a plane.

Identify how surface intersects planes $z = 0, 1, 2, 3 \ldots$ as below:

$$z = x^2 + y^2$$
put these together:

\[ z = x^2 + y^2 \]

note: intersection with zy plane is

note: this is basically the only general sketching technique available:

"taking traces" (z traces here)

note: understand technique, but don't memorize special surface names/families for now. This example serves as model for most others.

discuss: x traces? or y traces? easy to identify but hard to put together.
(Vectors): Def. a vector \( \vec{V} \) in \( \mathbb{R}^2 \), is a pair (ordered):

\[
\vec{V} = (a, b)
\]

represented by any arrow in \( \mathbb{R}^2 \) with components \( a, b \) as:

- If base pt. is \( P = (x_0, y_0) \), and pt. \( Q = (x, y) \) then
  
  \[ \vec{V} = (x - x_0, y - y_0) \]
  and write \[ \vec{V} = PQ \]

- Length of \( \vec{V} \) is defined as
  
  \[ \| \vec{V} \| = \sqrt{a^2 + b^2} \]
Def: a vector \( \mathbf{v} \) in \( \mathbb{R}^3 \), is an ordered triple
\[ \mathbf{v} = (a, b, c) \]

Vector arithmetic:
- Addition: \( \mathbf{v} + \mathbf{w} = (a + a', b + b', c + c') \)
- Scalar multiply: \( c\mathbf{v} = (ca, cb, cc) \)

\( \parallel \) parallel from law
\[ \frac{1}{2} \]
we say \( \vec{V}, \vec{W} \) parallel if \( \vec{V} = c \vec{W} \) some \( c \neq 0 \)

\( \vec{V}, \vec{W} \) same direction if \( \vec{V} = c \vec{W} \) some \( c > 0 \)

\( \vec{V} \) a unit vector if \( ||\vec{V}|| = 1 \)

Note: \( \vec{V}_{\text{unit}} = \frac{\vec{V}}{||\vec{V}||} = \frac{1}{||\vec{V}||} \vec{V} \) is always a unit vector with same direction as \( \vec{V} \). Verify!

see properties of vector arithmetic in books.

1) \( \vec{V} + \vec{W} = \vec{W} + \vec{V} \)
2) \( c(\vec{V} + \vec{W}) = c\vec{V} + c\vec{W} \)