Continue your result:

\[ \phi \phi \phi = \psi \psi \psi \]

\[ \psi = \phi \phi \phi \]

\[ \phi \phi \phi = \psi \psi \psi \]

\[ \psi = \phi \phi \phi \]

\[ \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \phi \phi \phi \psi \psi \psi \psi \psi \]

\[ \psi = \phi \phi \phi \]

\[ \phi \phi \phi = \psi \psi \psi \]

\[ \psi = \phi \phi \phi \]

\[ \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \phi \phi \phi \psi \psi \psi \psi \psi \]

\[ \psi = \phi \phi \phi \]

\[ \phi \phi \phi = \psi \psi \psi \]

\[ \psi = \phi \phi \phi \]

\[ \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \phi \phi \phi \psi \psi \psi \psi \psi \]

\[ \psi = \phi \phi \phi \]

\[ \phi \phi \phi = \psi \psi \psi \]

\[ \psi = \phi \phi \phi \]

\[ \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \phi \phi \phi \psi \psi \psi \psi \psi \]

\[ \psi = \phi \phi \phi \]

\[ \phi \phi \phi = \psi \psi \psi \]

\[ \psi = \phi \phi \phi \]

\[ \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \phi \phi \phi \psi \psi \psi \psi \psi \]

\[ \psi = \phi \phi \phi \]

\[ \phi \phi \phi = \psi \psi \psi \]

\[ \psi = \phi \phi \phi \]

\[ \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \phi \phi \phi \psi \psi \psi \psi \psi \]

\[ \psi = \phi \phi \phi \]

\[ \phi \phi \phi = \psi \psi \psi \]

\[ \psi = \phi \phi \phi \]

\[ \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \phi \phi \phi \psi \psi \psi \psi \psi \]

\[ \psi = \phi \phi \phi \]

\[ \phi \phi \phi = \psi \psi \psi \]

\[ \psi = \phi \phi \phi \]

\[ \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \phi \phi \phi \psi \psi \psi \psi \psi \]

\[ \psi = \phi \phi \phi \]

\[ \phi \phi \phi = \psi \psi \psi \]

\[ \psi = \phi \phi \phi \]

\[ \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \phi \phi \phi \psi \psi \psi \psi \psi \]

\[ \psi = \phi \phi \phi \]

\[ \phi \phi \phi = \psi \psi \psi \]

\[ \psi = \phi \phi \phi \]

\[ \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \phi \phi \phi \psi \psi \psi \psi \psi \]

\[ \psi = \phi \phi \phi \]

\[ \phi \phi \phi = \psi \psi \psi \]

\[ \psi = \phi \phi \phi \]

\[ \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \phi \phi \phi \psi \psi \psi \psi \psi \]

\[ \psi = \phi \phi \phi \]

\[ \phi \phi \phi = \psi \psi \psi \]

\[ \psi = \phi \phi \phi \]

\[ \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \phi \phi \phi \psi \psi \psi \psi \psi \]

\[ \psi = \phi \phi \phi \]

\[ \phi \phi \phi = \psi \psi \psi \]

\[ \psi = \phi \phi \phi \]

\[ \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \phi \phi \phi \psi \psi \psi \psi \psi \]

\[ \psi = \phi \phi \phi \]

\[ \phi \phi \phi = \psi \psi \psi \]

\[ \psi = \phi \phi \phi \]

\[ \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \phi \phi \phi \psi \psi \psi \psi \psi \]

\[ \psi = \phi \phi \phi \]

\[ \phi \phi \phi = \psi \psi \psi \]

\[ \psi = \phi \phi \phi \]

\[ \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \phi \phi \phi \psi \psi \psi \psi \psi \]

\[ \psi = \phi \phi \phi \]

\[ \phi \phi \phi = \psi \psi \psi \]

\[ \psi = \phi \phi \phi \]

\[ \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \phi \phi \phi \psi \psi \psi \psi \psi \]

\[ \psi = \phi \phi \phi \]

\[ \phi \phi \phi = \psi \psi \psi \]

\[ \psi = \phi \phi \phi \]

\[ \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \phi \phi \phi \psi \psi \psi \psi \psi \]

\[ \psi = \phi \phi \phi \]

\[ \phi \phi \phi = \psi \psi \psi \]

\[ \psi = \phi \phi \phi \]

\[ \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \phi \phi \phi \psi \psi \psi \psi \psi \]

\[ \psi = \phi \phi \phi \]

\[ \phi \phi \phi = \psi \psi \psi \]

\[ \psi = \phi \phi \phi \]

\[ \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \phi \phi \phi \psi \psi \psi \psi \psi \]

\[ \psi = \phi \phi \phi \]

\[ \phi \phi \phi = \psi \psi \psi \]

\[ \psi = \phi \phi \phi \]

\[ \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \phi \phi \phi \psi \psi \psi \psi \psi \]

\[ \psi = \phi \phi \phi \]

\[ \phi \phi \phi = \psi \psi \psi \]

\[ \psi = \phi \phi \phi \]

\[ \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \phi \phi \phi \psi \psi \psi \psi \psi \]

\[ \psi = \phi \phi \phi \]

\[ \phi \phi \phi = \psi \psi \psi \]

\[ \psi = \phi \phi \phi \]

\[ \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \phi \phi \phi \psi \psi \psi \psi \psi \]

\[ \psi = \phi \phi \phi \]

\[ \phi \phi \phi = \psi \psi \psi \]

\[ \psi = \phi \phi \phi \]

\[ \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \phi \phi \phi \psi \psi \psi \psi \psi \]

\[ \psi = \phi \phi \phi \]

\[ \phi \phi \phi = \psi \psi \psi \]

\[ \psi = \phi \phi \phi \]

\[ \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \phi \phi \phi \psi \psi \psi \psi \psi \]

\[ \psi = \phi \phi \phi \]

\[ \phi \phi \phi = \psi \psi \psi \]

\[ \psi = \phi \phi \phi \]

\[ \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \phi \phi \phi \psi \psi \psi \psi \psi \]

\[ \psi = \phi \phi \phi \]

\[ \phi \phi \phi = \psi \psi \psi \]

\[ \psi = \phi \phi \phi \]

\[ \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \phi \phi \phi \psi \psi \psi \psi \psi \]

\[ \psi = \phi \phi \phi \]

\[ \phi \phi \phi = \psi \psi \psi \]

\[ \psi = \phi \phi \phi \]

\[ \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \phi \phi \phi \psi \psi \psi \psi \psi \]

\[ \psi = \phi \phi \phi \]

\[ \phi \phi \phi = \psi \psi \psi \]

\[ \psi = \phi \phi \phi \]

\[ \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \phi \phi \phi \psi \psi \psi \psi \psi \]

\[ \psi = \phi \phi \phi \]

\[ \phi \phi \phi = \psi \psi \psi \]

\[ \psi = \phi \phi \phi \]

\[ \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \phi \phi \phi \psi \psi \psi \psi \psi \]

\[ \psi = \phi \phi \phi \]

\[ \phi \phi \phi = \psi \psi \psi \]

\[ \psi = \phi \phi \phi \]

\[ \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \phi \phi \phi \psi \psi \psi \psi \psi \]

\[ \psi = \phi \phi \phi \]

\[ \phi \phi \phi = \psi \psi \psi \]

\[ \psi = \phi \phi \phi \]

\[ \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \phi \phi \phi \psi \psi \psi \psi \psi \]
\[ \phi = \arccos \left( \frac{-1}{2} \right) \]

\[ \phi = \arccos \left( \frac{-1}{2} \right) \]

\[ \phi = \arccos \left( \frac{-1}{2} \right) \]

\[ 1 = \phi \times \frac{\pi}{2} + \phi \cos \phi - \phi \sin \phi \]

\[ 1 = \sqrt{\phi^2 \times \frac{\pi}{2} + (1 - \phi \cos \phi)} \]

\[ 1 = \phi \times \frac{\pi}{2} + \phi \cos \phi - \phi \sin \phi \]

\[ 1 = \sqrt{\phi^2 \times \frac{\pi}{2} + (1 - \phi \cos \phi)} \]

\[ 2 - 2\phi - 1 = 2(1 - \phi) \]

\[ 2 - 2\phi - 1 = 2(1 - \phi) \]

\[ 2h - 2x - 1 = 2(1 - \phi) \]

\[ 2h - 2x - 1 = 2(1 - \phi) \]

\[ \frac{2h - 2x - 1}{1 + 1} = (1 - \phi) \]

\[ \frac{2h - 2x - 1}{1 + 1} = (1 - \phi) \]

\[ x \neq \frac{h + \sqrt{h^2 + x^2}}{2} \]

\[ x \neq \frac{h + \sqrt{h^2 + x^2}}{2} \]

\[ 2h - 2x - 1 \neq 1 \]

\[ 2h - 2x - 1 \neq 1 \]

\[ 2 - 2h - 1 \neq 0 \]

\[ 2 - 2h - 1 \neq 0 \]

\[ 2x - 1 \neq 1 \]

\[ 2x - 1 \neq 1 \]

\[ \text{Concentric to spherical case:} \]

\[ \text{Concentric to spherical case:} \]

\[ \phi' \neq \frac{\pi}{2} \]

\[ \phi' \neq \frac{\pi}{2} \]
Easily. Can get idea of shape

In order to deduce know it is

Note: \( \int_0^\pi \cos \phi \, d\phi = 2 \cos \phi \)

\( \int_0^\pi \cos \phi \, d\phi = 2 \cos \phi \)

Calcutt, until the time to repeat steps above

Conversely, count angle to center (3)

\( \int_0^\pi \cos \phi \, d\phi = 2 \cos \phi \)

\( \int_0^\pi \cos \phi \, d\phi = 2 \cos \phi \)

\[ \int_0^\pi \cos \phi \, d\phi = 2 \cos \phi \]
### Rules Governing Examinations

- Examination materials from the examination room without permission shall not be removed.
- Candidates must not destroy or mutilate any examination notes.
- No food or drink is allowed in the examination room.
- Examination materials and devices other than those authorized by the examination shall not be used.
- Examination materials shall be immediately destroyed after the examination.
- Candidates are not permitted to ask questions of the examiners.
- Answers to all questions must be shown on the examination paper.
- No correction or modification of answers is allowed.
- No electronic devices or calculators are allowed.

### Table

<table>
<thead>
<tr>
<th>Total</th>
<th>30</th>
<th>11</th>
<th>10</th>
<th>7</th>
<th>13</th>
<th>6</th>
<th>3</th>
<th>2</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>8</td>
<td>7</td>
<td>6</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

### Special Instructions:

- Closed book examination.
- No notes, aids, or calculators.

---

*Note: The text above is a partial transcription of the image.*

---

*Note: This document contains a table with examination rules and instructions.*

---

*Note: The table includes examination scores.*

---

*Note: The document is part of a final examination for MATH 200 at The University of British Columbia.*
Consider the integral \( \int_{\mathcal{D}} \int_{\mathcal{R}} \int_{\mathcal{S}} = I \)
Problem 5.2 of Vector Calculus

(a) Calculate the volume of the solid enclosed by the given surfaces.

(b) Evaluate the integral 
\[ \iiint_V \frac{1}{r} \, dV \]

(c) Evaluate the surface integral 
\[ \iint_S \mathbf{F} \cdot d\mathbf{S} \]

where \( \mathbf{F} = \mathbf{F}(x, y, z) \) is a vector field and \( S \) is the surface.

(d) Consider the region \( D \) in the first octant defined by \( x^2 + y^2 + z^2 \leq a^2 \) and \( z \leq x + y \).

Let \( f(x, y, z) \) be a scalar function defined on \( D \).

(e) Evaluate the line integral 
\[ \oint_C f(x, y, z) \, ds \]

where \( C \) is a closed curve lying in the plane \( x + y + z = a \).
\[
\begin{align*}
\frac{3}{1} - \frac{3}{1} \cos \frac{3}{1} &= \\
\int_{0}^{1} \left[ \frac{3}{1} - \frac{3}{1} \cos \frac{3}{1} \right] \, dy = \\
\end{align*}
\]
\(s(x, y) = \pi y - y^3, x = 0, y = 3\)

**(a) Evaluate \(I\):**

\[
I = \int_{0}^{1} \int_{0}^{3} s(x, y) \, dy = \frac{ax^2}{4} - \frac{ax}{2} + \frac{a}{4}
\]

Let \(L \in \mathbb{R}\) be a point in the plane that satisfies these integrals.

### Problem 6

Sketch the corresponding region of integration in the plane, label your sketch sufficiently.
The temperature in the plane is given by $T(x,y) = y^2 + x^2 e^{-x}$. To find the warmest and coolest points on the circle $x^2 + y^2 = 1$, we must solve the following system of equations:

1. \[ 0 = \frac{\partial T}{\partial x} = 2x e^{-x} + 2x^2 e^{-x} \]
2. \[ 0 = \frac{\partial T}{\partial y} = 2y \]
3. \[ 0 = \frac{\partial^2 T}{\partial x^2} = 2e^{-x} + 2x^2 e^{-x} - x e^{-x} \]

The warmest point is $(0,0)$, and the coolest points are $(0,10)$ and $(0,-10)$.
\[ \begin{align*}
\frac{\partial f}{\partial x} &= -f_x \\
\frac{\partial f}{\partial y} &= -f_y
\end{align*} \]

\[ \Theta(x, y) = \Theta(x, y) \]

\[ \frac{\partial \Theta}{\partial x} + \frac{\partial \Theta}{\partial y} = 0 \]

\[ \frac{\partial}{\partial x} \left( \frac{\partial f}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{\partial f}{\partial y} \right) = 0 \]

**Tips:**
- (a) Fill blanks below in terms of functions depending on \( x \), \( y \), and \( \Theta \), and partial derivatives.
- (b) Let \( g(x, y) \) be another function satisfying \( \Theta \).

Suppose \( f(x, y) \) is twice differentiable (with \( \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x} \)) and \( x = r \cos \theta \) and \( y = r \sin \theta \).
(e) The coolest point on the solid disk $x^2 + y^2 \leq 100$ is $(0,0)$.

Tips (i) To find the critical points of $f(x,y)$ we must solve the following system:

\[
\begin{align*}
\frac{\partial f}{\partial x} &= x^2 + xy = 0 \\
\frac{\partial f}{\partial y} &= x^2 + y = 0
\end{align*}
\]

Take the same temperature function as in part (a).

Tips (ii) By solving the above system we conclude the critical points are:

$$ (0,0), (0, -2) $$

 questões

\[
\begin{align*}
\frac{\partial f}{\partial x} &= x^2 + xy = 0 \\
\frac{\partial f}{\partial y} &= x^2 + y = 0
\end{align*}
\]

Take the same temperature function as in part (a).
3. Consider the functions $F(x, y, z) = x^2 + y^2 z$ and $G(x, y, z) = 3x - y + 4z$. You are standing at the point $P(0, 1, 2)$. If you jump from $P$ to $Q(0, 0, 1, 8)$, then the amount by which $F$ changes is approximately 2:

$$\Delta F = (y_2 - y_1) + (z_2 - z_1) + 12(-2.4) = 3.4 - 2.4$$

(Use linear approximation.)

5 pts (a) If you jump from $P$ to $Q(0,1,0,1,8)$, then the amount by which $F$ changes is approximately:

$$\Delta F = (y_2 - y_1) + (z_2 - z_1) + 12(-2.4)$$

5 pts (b) If you jump from $P$ in the direction along which $G$ increases most rapidly, then $G$ will increase/decrease (circle one and explain below).

$$\nabla G = \langle 3, 2, 4 \rangle$$

$$G(\mathbf{c}) = 3 \cdot a + 2 \cdot b + 4 \cdot c$$

5 pts (c) You jump from $P$ in a direction $(a, b, c)$, along which rate of change of $F$ and $G$ are both zero. An example of such a direction is $(a, b, c) = \langle 2, -2, -1 \rangle$ (not be unit vector).

$$V_{F}(P) \cdot V = 0 \Rightarrow 3 \cdot a + 2 \cdot b + 4 \cdot c = 0$$

$$V_{G}(P) \cdot V = 0 \Rightarrow 2 \cdot a - b + c = 0$$

$$\Rightarrow a = 4$$

$$b = 3a - 4c$$

$$\Rightarrow c = 1$$

$$\Rightarrow b = 8$$

$$V = \sqrt{v_{x}^{2} + v_{y}^{2} + v_{z}^{2}}$$

$$\Rightarrow V = 12 - 4$$

could also use

$$V = \sqrt{v_{x}^{2} + v_{y}^{2} + v_{z}^{2}}$$

$$\Rightarrow V = 12 - 4$$
below choose the correct-proof.

(i) The first order partial derivative $f_x(x,y)$ is zero (chole one).
(ii) The second order partial derivative $f_{xx}(x,y)$ is positive (chole one).
(iii) Has a critical point at $(2,2)$.

For each statement, below choose those whose graphs intersect or cross with the hyperbolic paraboloid $z = x^2 - y^2$. The direction below shows those whose graphs intersect or cross with the hyperbolic paraboloid $z = x^2 - y^2$. None of the other choices. You may assume that a local maximum occurs at point $I$.

$\text{negative}$

$\text{negative}$

$\text{positive}$

$\text{positive}$

$\text{saddle point}$

$\text{saddle point}$

$\text{zero}$

$\text{zero}$

(explain the location where the surface intersects the $x$-axis; you would choose both $x$ and $y$, but explain the location where the surface intersects the $y$-axis; you would choose both $x$ and $y$, but...
1. Let \( A = (0, 2, 2), B = (2, 2, 2), C = (5, 2, 1) \).

(a) The line which contains \( A \) and is perpendicular to the triangle \( ABC \) has parametric equations:

\[
\begin{align*}
\mathbf{r} &= \mathbf{r}_0 + t\mathbf{v} \\
\mathbf{r} &= (0, 2, 2) + t(2, 0, -2) \\
\mathbf{r} &= (2t, 2, 2-2t)
\end{align*}
\]

(b) The set of all points \( P \) such that \( \overrightarrow{PA} \) is perpendicular to \( \overrightarrow{PB} \) form a Plane.

(c) A light source at the origin shines on triangle \( ABC \) making a shadow on the plane. If \( x + y + z = 32 \) (see diagram), then \( A = (0, 0, 0), B = (2, 2, 2), C = (5, 2, 1) \).

\[
\begin{align*}
&0 + 0 + z = 32 \\
&z = 32 \\
&\Rightarrow y + z = 32 \\
&y + 32 = 32 \\
&y = 0
\end{align*}
\]
\[ 0 = (1-z)(1-y)(1) + \frac{b}{x-z} + \frac{b}{x-y} \]

\[ z = 1 \]

\[ x = 2 \]

\[ y = 1 \]

\[ z = \frac{3 + e^2 + e^k + e^l}{2} \]

At the point (2,1,1),

\[ 0 = \frac{3 + e^2 + e^k + e^l}{2} \]

(c) Find the tangent plane to

\[ \begin{cases} 
1 - t &= z \\
1 + t &= y \\
1 &= x 
\end{cases} \]

\[ \begin{cases} 
x &= 1 \\
y &= 2 \\
z &= 3 
\end{cases} \]

(b) Find the parametric equation for the line of intersection of the planes

\[ x = z - k - x \quad \text{and} \quad y = k + x + z \]

\[ x = 1 \]

\[ y = 1 \]

\[ z = 1 \]

\[ x = c - 1 \]

\[ y = 3 - c \]

\[ z = 2 \]

\[ \frac{9 + b}{13 - c} \]

Distance from the above plane. Your answers should be in the following form: \( x + 2y - 4z = 0 \).

(a) Consider the plane \( x + 2y - 4z = 3 \). Find all parallel planes that are distance 2 from

[5]
<table>
<thead>
<tr>
<th>Question</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>89</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>8</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td>8</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
</tr>
</tbody>
</table>

**Student Conduct during Examinations**

- Candidate must follow any additional examination rules or directions communicated by the examiner.
- Candidate must not disturb any candidate in the examination room.
- Candidate must not carry any objects into the examination room.
- Candidate must not move from the examination desk. Any communication material must be handed in at the conclusion of the examination.
- Candidate must not leave the examination room without permission from the examiner.
- Candidate must not damage or leave any examination material.
- Candidate must not copy or write down the questions or any part of the examination.
- Candidate must not use any electronic devices during the examination.
- Candidate must not exchange any materials with other candidates.
- Candidate must not communicate with other candidates who are listening to the examination.
- Candidate must not leave the examination desk without permission from the examiner.
- Candidate must not leave the examination room without permission from the examiner.

**Student Instructions**

- Show your work for all problems. No books, notes, or calculators are allowed. Show your work for all problems.

**Student Details**

- Student Number
- Signature
- First Name
- Last Name

**Time**

- 2:50 hours

**Mathematics 200**

- Exam Date: December 17, 2017
- The University of British Columbia
--- Some of 3c 2016.

\[ e = \left( \frac{\partial F}{\partial y} \right)_T \]

\[ 0 = \left( \frac{\partial F}{\partial y} \right)_T \]

So that:

\[ \frac{\partial F}{\partial y} \]

\[ \text{see } 2016 \text{ Feb. } g = \text{just west } U \leq \text{g} \leq c \]

zero in this direction.

should the bee fly so that the rate of change of \( \frac{\partial F}{\partial y} \) and of \( S(x, y, z) \) are both

![Image](image-url)

Let \( S(x, y, z) = x + 2y \). A bee starts flying at \( P \); along which unit vector direction

\[ 5.5 = 5 + 1 + 0.6 \]

\[ 5 + 1 - (1 - 1) + 2(0) + 3(2) \]

\[ \left( \begin{array}{c} 2 \frac{d(x)}{dt} + 0 \frac{d(t)}{dt} + 1 \frac{d(t)}{dt} \\ 1 \frac{d(t)}{dt} + 1 \frac{d(t)}{dt} + 1 \frac{d(t)}{dt} \end{array} \right) \]

(i) Use linear approximation of \( f \) at the point \( P \) to approximate \( f(1.9, 1.7, 1.2) \).

\[ \frac{\Delta f}{\Delta t} = \frac{3}{3} \]

\[ \frac{3}{3} \]

\[ \left( \begin{array}{c} 2 \frac{d(x)}{dt} + 2 \frac{d(t)}{dt} \\ 1 \frac{d(t)}{dt} + 1 \frac{d(t)}{dt} \end{array} \right) \]

\[ \text{in this direction?} \]

\[ \text{And } \frac{\partial F}{\partial y} = 0, \text{ What is the rate of change of } \frac{\partial F}{\partial y} \text{ ?} \]

\[ \text{At } P, \text{ the unit vector pointing towards the point} \]

\[ (d) \frac{d}{dt} = 1, y \frac{d}{dt} = 2, \text{ and } (d) \frac{d}{dt} = 3 \text{.} \]

\[ \text{A function } f \text{ at } P \]
\[ A = -1 \]

\[ \frac{d}{dx} f(x) = -w^t + \frac{d}{dx} = 0 \]

\[ = \int (u \cdot x + v \cdot y) \]

\[ = 13u \cdot x + 15u \cdot y \]

\[ \Rightarrow w^t + u^t = 13u \cdot x + 15u \cdot y \]

\[ w^t \cdot 0 + u^t \cdot 15u \cdot y \]

\[ = 15u \cdot y \]

(b) Suppose \( u^t + w^t = 0 \). For what constant \( A \) will \( u^t = Aw^t \)?

\[ \frac{d}{dx} (x^2 + 12x + 9) \]

\[ = (2x + 12) \cdot 2 + (x^2 + 12x + 9) \cdot 3 \]

\[ = 2x^2 + 12x + 9 \]

\[ w^t = \frac{d}{dx} (x^2 + 12x + 9) \]

\[ = 2x + 12 \]

\[ \Rightarrow w^t = \frac{d}{dx} (x^2 + 12x + 9) \]

\[ \Rightarrow \frac{d}{dx} (x^2 + 12x + 9) = 2x + 12 \]

(a) Find \( w^t \) in terms of \( u^t \) and \( v^t \). You can assume that \( w^t \) and \( v^t \) (you can assume that \( w^t \) and \( v^t \))

\[ (x+y)^n = \frac{d}{dx} (x+y)^n \]

\[ \Rightarrow \frac{d}{dx} (x+y)^n = (x+y)^n (\frac{d}{dx} x + \frac{d}{dx} y) \]

3. Let \( m(x) = (2x^3 - 3x^2 - 21) \) for some twice differentiable function \( n = x^2 \).
Find and classify the critical points of $f(x, y) = 3x^2y + y^3 - 3x^2 - 3y^2 + 4$.

$f_x = 6xy - 6x = 0$

$f_y = 3x^2 + 3y^2 - 6y = 0$

$x = 0$ or $y = 0$

$y = 0$ or $y = 1$

Now apply 2nd Der Test to classify, use $D_{(0,0)} = f_{xx}f_{yy} - f_{xy}^2$.
5. Use Lagrange multipliers to find the minimum and maximum values of \((x + y)/e^x\), subject to \(y^2 + x^2 = 6\).

\[
\begin{align*}
\frac{\partial}{\partial x} (x^2 + y^2) &= 2x \\
\frac{\partial}{\partial y} (x^2 + y^2) &= 2y \\
\frac{\partial}{\partial \lambda} (x^2 + y^2) &= 0
\end{align*}
\]

\[
\begin{align*}
x &= 2 \\
y &= 2 \\
\lambda &= 1
\end{align*}
\]

\[
f((1, 2, 1)) = \left(\frac{2}{e^2}\right)^2 \text{ New Value}
\]

\[
f((-1, -2, -1)) = \left(\frac{-2}{e^2}\right)^2 \text{ New Value}
\]
(c) Compute the integral in the case \( f(x, y) = y^2 - x^2 \). Then express the integral as an iterated integral corresponding to the order \( dy \, dx \). Sketch \( D \).

\[ \iint_D f(x, y) \, dx \, dy \]

(b) Express the integral as an iterated integral corresponding to the order \( dx \, dy \). Sketch \( D \).

[6] Consider the domain \( D \) above the \( x \)-axis and below parabola \( y = 1 - x^2 \) in the \( xy \)-plane.

Dec. 17, 2015
as three different iterated integrals corresponding to the orders of integration: (a) \( \int \int \int \) \\
(b) \( \int \int \int \) \\
(c) \( \int \int \int \) \\

Express the integral as the plane \( z = 1 \) below the plane \( z = \frac{1}{2} \) and above the plane \( z = \frac{1}{2} \).

\[ \int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{-\sqrt{1-z^2}}^{\sqrt{1-z^2}} f(x, y, z) \, dz \, dy \, dx \]