10.1

- 3-dim coord systems in space
- distance between pts.
- simple surfaces/equations in space

3 dim coordinates system

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std:
"right hand system/frame"
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3 mutually perpendicular directed line segments emanating from an origin in space.
label as x, y, z
Given any ordered triple \((a, b, c)\), a point in space is then determined by:

Starting from \(O\) then:

- Go over \(a\) units in \(x\) dir.
- Go over \(b\) units in \(y\) dir.
- Then go over \(c\) units in \(z\) dir.

\(\therefore\)
e) Plot the points:

- $P = (1, 5, 3)$
- $Q = (0, 4, 2)$
- $R = (0, 0, 3)$

Rectangular box with dimensions $1 \times 5 \times 3$ units not oriented.
Q: What do you notice about \( P, Q \)?

Q: Does it matter what order we travel in the coordinate directions to get to \( P \)? Say \((x \text{ then } y \text{ then } z)\) vs \((y \text{ then } z \text{ then } x)\)?

Q: Speaking of frames, consider the standard `left hand frame` at origin.

A: Looks like same pt!

A: No.

A:
The distance between two points $P(a_1, b_1, c_1)$ and $Q(a_2, b_2, c_2)$ is given by the formula:

$$\text{dist.}(P, Q) = \sqrt{(a_1 - a_2)^2 + (b_1 - b_2)^2 + (c_1 - c_2)^2}$$
Proof: use a specific example to illustrate

\[ P = (1, 2, 1), \quad Q = (5, 7, 6) \]

\[ C^2 = A^2 + B^2 \quad \checkmark \]

\[ = a^2 + b^2 + B^2 \quad \checkmark \]

\[ = (5-1)^2 + (7-2)^2 + (6-1)^2 \]
Simple Surfaces / equations

The set of all pts. \((x, y, z)\) in space which satisfy the equation

\[
(x-a)^2 + (y-b)^2 + (z-c)^2 = r^2
\]

can be described as:

\((a, b, c, r)\) are some fixed constants.
(8)

\[ (x+1)^2 + y^2 + z^2 = 2 \]

\[ x^2 + 2x + y^2 + z^2 = 1 \]

Sketch the set of all points in space.
general principle:

The set of all pts \((x, y, z)\) in space satisfying a single equation

\[ F(x, y, z) = 0 \]

will be a surface.

spheres are a typical example. Here's more!

Here are some other examples...
e) sketch surfaces of these equations:

\[ x + y = 1 \]

\[ z = 2 \]

\[ z = \sin y \]

\[ x^2 + x = 1 \]

f) a plane in space.
(iii)

(iv) \( x^2 + y^2 = 1 \)

"a wavy sheet"

"cylinder"
eg) Find equation of surface of all pts $P$ which are equidistance to $Q = (1, 1, 1)$ and $R = (0, 0, 0)$

(think of what this looks like)

Surface of all $P = (x, y, z)$

$\Rightarrow \text{dist}(P, Q) = \text{dist}(R, P, R)$

$\Rightarrow \sqrt{(x-1)^2 + (y-1)^2 + (z-1)^2} = \sqrt{x^2 + y^2 + z^2}$

$\Rightarrow (x-1)^2 + \cdots = x^2 + \cdots$

$\Rightarrow (x^2 - 2x + 1) + \cdots = x^2 + \cdots$
Keep this in mind. Stay focused. Stay tuned.

(Up to multiple integration)

Techniques in the course

A: These involve most of the

on surfaces)
higher and low pts
(examples &

Q: Are there any high/low pts on S?

Q: In what direction at P is S the steepest?

Q: Around the point P = (1,1) say if

Q: Can you describe S, even a small portion of it

2x^2 + yz = 3

(3)

Consider surface S in space given by
Vectors 10.2

* def
* length, direction
* arithmetic

The most fundamental concept in our discussion of space is the point (position)
The second most is the vector (velocity/direction...
Def

A vector \( \vec{V} \) in \( \mathbb{R}^3 \) is an ordered triple

\[
\vec{V} = \langle a, b, c \rangle
\]

which we realize/identify with an ‘arrow’ in space with ‘components’ \( a, b, c \)

\[\begin{align*}
\text{if base pt. } & P = (x_1, y_1, z_1) \\
\text{end pt. of } & Q = (x_2, y_2, z_2) \\
\text{then } & a = x_2 - x_1 \\
& b = y_2 - y_1 \\
& c = z_2 - z_1
\end{align*}\]

and also denote \( \vec{V} = PQ \)
A vector \( \mathbf{v} \) in plane.

Notice, base pt. \( P \) can be anything, long point.

What physical things can be described by vectors in \( \mathbb{R}^2 \)?

What is base pt. is irrelevant.??

\( \mathbf{v} \) is wind force fields

\( \mathbf{v} \) is NE wind with speed \( \sqrt{2} \) say, ??

Could be SE wind with speed \( \sqrt{2} \) say, ??

exact analogous definition for

\( a \odot b = a \times \mathbf{b} \in \mathbb{R}^2 \)
Given vectors \( \vec{v} = \langle a, b, c \rangle \) and \( \vec{w} = \langle a_2, b_2, c_2 \rangle \):

- Length of \( \vec{v} \) is \( \|\vec{v}\| = \sqrt{a^2 + b^2 + c^2} \)

- \( \vec{v}, \vec{w} \) said to have if same direction:

- \( \vec{v} + \vec{w} := \langle a_1 + a_2, b_1 + b_2, c_1 + c_2 \rangle \) (addition)

- \( \alpha \vec{v} := \langle \alpha a, \alpha b, \alpha c \rangle \) (scaling)

- \( \cdot \cdot \cdot \) LATER!
Vectors in nature actually do combine in this way.

Geometrically, addition looks like

\[ v + w = v + w \]

Vectors can be added and factored.

Example:

\[ v + w = v + w \]

Illustration of vector addition

See properties of vector arithmetic in Text
Typically, we denote:

\[ \vec{i} = \langle 1, 0, 0 \rangle \]
\[ \vec{j} = \langle 0, 1, 0 \rangle \]
\[ \vec{k} = \langle 0, 0, 1 \rangle \]

and sometimes use notation:

\[ \vec{v} = \langle a, b, c \rangle = a\vec{i} + b\vec{j} + c\vec{k} \]

* Given pt. \( P = (a, b, c) \), its position vector in \( \vec{v} = \overrightarrow{OP} \) where \( O \) is the origin.
eg) find vector length $\mathbf{w}$ with same direction as $\mathbf{w} = \langle 1, 5, 7 \rangle$

eg) repeat for lengths 5, 10, 7

eg) write $\mathbf{c}$, $\mathbf{d}$ in terms of $\mathbf{a}, \mathbf{b}$.
(i) We are finding \( \lambda \) so that \((\lambda > 0)\)

\[
\vec{v} = \langle \lambda, 1, \lambda, 5, \lambda, 7 \rangle
\]

has length 1.

Do as: 

\[
\|\vec{w}\| = \sqrt{1^2 + 5^2 + 7^2}
\]

\[
= \sqrt{1 + 25 + 49}
\]

\[
= \sqrt{75}
\]

Then \( \vec{v} = \frac{1}{\sqrt{75}} \vec{w} \) is vector desired. (check!)

(generally, \( \frac{\vec{v}}{\|\vec{v}\|} \) is unit vector, same dir as \( \vec{v} \) !)

(ii) \( 5\vec{v}, 10\vec{v}, 7\vec{v} \ldots = \frac{5}{\sqrt{75}} \vec{w}, \frac{10}{\sqrt{75}} \vec{w}, \frac{7}{\sqrt{75}} \vec{w} \ldots \) (check!)
(iii) \[ 2 \vec{d} = -\vec{a} + \vec{b} \]
\[ \vec{d} = -\frac{1}{2} \vec{a} + \frac{1}{2} \vec{b} \]