1. (a) (2pts) Find the equation of the plane containing the points (1,1,1), (2,5,2), (2,1,2) (express as \(ax + by + cz = d\)).

\[
\begin{align*}
\langle 2 - 1, 5 - 1, 2 - 1 \rangle &= \langle 1, 4, 1 \rangle \\
\langle 2 - 2, 1 - 1, 2 - 1 \rangle &= \langle 0, 0, 1 \rangle \\
\end{align*}
\]

are a coplanar.

\[
\vec{u} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ -4 \end{bmatrix}
\]

\[4(x-1) - 4(2-1) = 0\]

\[
\begin{bmatrix} x-2 \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}
\]

(b) (2pts) Express the vector \(\vec{h}\) in terms of the vectors \(\vec{a}, \vec{b}, \vec{c}\).

(you may use that the projection of a vector \(\vec{u}\) onto a vector \(\vec{v}\) is given by \(\frac{\vec{a} \cdot \vec{b}}{\|\vec{v}\|^2} \vec{v}\))

\[
\vec{h} = \frac{\vec{c} \cdot (\vec{a} \times \vec{b})}{\| (\vec{a} \times \vec{b}) \|^2} (\vec{a} \times \vec{b})
\]

\(\) a parallellepiped

(c) (3pts) A bee flies in a straight line from the point \(A = (1,1,1)\) to the point \(B = (2,3,4)\), and somewhere in between it passes through the plane \(x + y + z = 6\) at \(P\). Find the coordinates of \(P\).

\[
\begin{align*}
x &= 1 + t \\
y &= 1 + 2t \\
z &= 1 + 3t
\end{align*}
\]

\[
\text{hits plane} \Rightarrow (1+t) + (1+2t) + (1+3t) = 6
\]

\[
\Rightarrow 3 + 6t = 6
\]

\[
\Rightarrow t = \frac{1}{2}
\]

\[
P = \left( 3, \frac{5}{2}, \frac{5}{2} \right)
\]
2 (a) (8 pts) Match the functions with their graphs and level curve plots by filling in the table below. You do not need to justify your answers.

<table>
<thead>
<tr>
<th>Function</th>
<th>Graph</th>
<th>Contour</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{x}{2}y \sin(\pi x)^2 )</td>
<td>A</td>
<td>(A)</td>
</tr>
<tr>
<td>( e^{-</td>
<td>x</td>
<td>} )</td>
</tr>
<tr>
<td>( 1 -</td>
<td>x</td>
<td>-</td>
</tr>
<tr>
<td>( \frac{-2xy}{1+x^2+y^2} )</td>
<td>E</td>
<td>(D)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Graph</th>
<th>Contour</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A)</td>
<td>(1)</td>
</tr>
<tr>
<td>(B)</td>
<td>(2)</td>
</tr>
<tr>
<td>(C)</td>
<td>(3)</td>
</tr>
<tr>
<td>(D)</td>
<td>(4)</td>
</tr>
<tr>
<td>(E)</td>
<td>(5)</td>
</tr>
</tbody>
</table>
3 a) (3pts) Let \( f(x, y) = \frac{1}{2} \ln(x^2 + y^2) \). Which of the following differential equations does \( f \) satisfy (circle all that apply)?

\[
\begin{align*}
& f_{xy} + 2f_yf_x = 0 \\
& (f_x)^2 + (f_y)^2 = 0
\end{align*}
\]

\[
\begin{cases}
  f_x = \frac{x}{x^2 + y^2} \\
  f_y = \frac{y}{x^2 + y^2} \\
  f_z = -\frac{2xy}{(x^2 + y^2)^2}
\end{cases}
\]

b) (3pts) At what points on the graph \( z = x^2 + xy - \frac{y^2}{2} - 3y \) is the tangent plane horizontal? (ie, when does tangent plane have the equation \( z = c \) for some constant \( c \))

This happens when

\[
\begin{align*}
  f_x &= 2x - y = 0 \\
  f_y &= x - y - 3 = 0
\end{align*}
\]

\[
\Rightarrow \begin{cases}
  2x + y = 0 \\
  x - y = 0
\end{cases}
\]

\[
\Rightarrow 3x = 3 \quad \Rightarrow \quad x = 1, y = -2
\]

- Tangent plane horizontal at \((1, -2, 3)\)
4. Let \( z = f(x, y) \) be defined implicitly by \( xz^3 + yz + x^2 = 2 \).

a) (4pts) Find \( z_x(0,1) \) and \( z_y(0,1) \)

\[
\begin{align*}
  &z^3 + xz^2 z_x + y z_x + 2 x = 0 \quad \Rightarrow \quad z_x = \frac{-z^3 - 2x}{3xz^2 + y} \quad (8) \\
  &x^3 z^2 z_y + 2y z_y = 0 \quad \Rightarrow \quad z_y = \frac{-2}{3xz^2 + y} = (2)
\end{align*}
\]

\[\text{Note:} \quad x = 0, \ y = 1 \quad \Rightarrow \quad z = 2\]

\[\text{Note:} \quad \text{could have used} \quad z_x = \frac{-F_x}{F_z}, \ z_y = \frac{-F_y}{F_z} \quad \text{to get same ans.}\]

\[\text{Note:} \quad \text{You should have used} \quad z_x = F_y, \ z_y = F_y \quad \text{to get very close answers, but is completely wrong if you use formulae and do not get credit!}\]

b) (2pts) Approximate \( f(0.2, 1.3) \) using the linear approximation of \( f(x, y) \) at an appropriate point.

\[
f(0.2, 1.3) \approx 2 - 8(0.2) - 2(1.3)
\]

c) (1pt) Give a normal vector for the tangent plane to the graph \( z = f(x, y) \) at the point \((0, 1, 2)\).

\[ \vec{N} = \langle 8, 2, 1 \rangle \]
5. At a given point in time a solid rectangular box has; its length equal to 1cm and changing at A cm/s; its width equal to 2cm and changing at B cm/s; and its height equal to 1cm and changing at C cm/s

a) (3 marks) At what rate is the surface area of the box changing at this time if A=7, B=9, C=11?

Let \[ S(l, w, h) = 2lw + 2wh + 2lh \]

Then \[ \frac{dS}{dt} = S_x \ell_t + S_w w_t + S_h h_t \]

\[ = 2(w+h) \cdot A + 2(l+h) \cdot B + 2(w+l) \cdot C \]

\[ = 6 \cdot 7 + 4 \cdot 9 + 6 \cdot 11 \text{ cm}^2/\text{s} \]

at the time in question

b) (2 marks) Find values for A, B, C (not all zero) so that the rate of change of the surface area is zero at this time.

Wait \[ \frac{dS}{dt} = 6A + 4B + 6C = 0 \]

\[ \Rightarrow \begin{cases} A = 4 \text{ works, } \sum \text{(many other answers)} \\ B = -\frac{11}{2} \\ C = 0 \end{cases} \]