Quiz 3

1. How small must $|z-i|$ be to ensure

$$|z^2+1| \leq 3 \quad ?$$

2. Let $f(x+iy) = x^2+iy^2$

a) Show that f is differentiable at $z=0$.

b) is $f(z)$ analytic on any open set ?

1. $|z^2+1| = |z-i||z+i| = |z-i|\,|z-i+i|$

$$\leq |z-i|\,(|z-i|+1)$$

$$= 3$$

$$\Rightarrow |z-i|^2+|z-i| = 3$$

$$\Rightarrow |z-i| = 1, \cancel{-2}$$

$\therefore$ $|z-i|$ must be ≤ 1

2. $$\lim_{\Delta x, \Delta y \to 0} \frac{f(x+\Delta x, y+\Delta y) - f(x,y)}{\Delta x + i\Delta y}$$

$$= \lim_{---} \frac{(x+\Delta x)^2 + i(y+\Delta y)^2 - f(x,y)}{\Delta x + i\Delta y}$$

$$= \lim_{---} \frac{(2x\Delta x + \Delta x^2) + i(2y\Delta y + \Delta y^2)}{\Delta x + i\Delta y}$$

a) above limit $= 0$ when $x = y = 0$

since $$\left|\frac{\Delta x^2 + i\Delta y^2}{\Delta x + i\Delta y}\right| \leq \frac{\Delta x^2}{|\Delta x + i\Delta y|} + \frac{\Delta y}{|\Delta x + i\Delta y|}$$

$$\leq \Delta x + \Delta y \to 0 \quad \text{as } \Delta x, \Delta y \to 0$$

$\therefore$ f diff. at $z=0$

b) above limit $= \begin{cases} 2x & \text{if } \Delta y = 0 \\ 2y & \text{if } \Delta x = 0 \end{cases}$

$\therefore$ f not diff when $y \neq x$

$\therefore$ f cannot be analytic on any open set