

HOMEWORK #3

$$2.2 \ #9 \quad \left| \frac{1}{z} - i \right| = \left| \frac{1}{z}(-i)(z+i) \right|$$

$$= \frac{1}{|z|} |z+i| \leq \frac{1}{|z+i|} |z+i| \leq \frac{1}{|1-|z||} |z+i|$$

by reverse triangle
inequality

Thus given $\epsilon > 0$, choose $\delta = \min\left(\frac{1}{2}, \frac{\epsilon}{2}\right)$. Then

$$|z+i| < \delta \Rightarrow \left| \frac{1}{z} - i \right| < \frac{1}{|1-\frac{1}{2}|} \cdot \frac{\epsilon}{2} = 2 \cdot \frac{\epsilon}{2} = \epsilon$$

$$\therefore \lim_{z \rightarrow -i} \frac{1}{z} = i$$

#17 No limit!

$$\left\{ \begin{array}{l} \lim_{x \rightarrow 0} f(x+iy) = \lim_{x \rightarrow 0} \frac{x^2}{x^2 + 2i} + 2i = 1+2i \\ \lim_{y \rightarrow 0} f(0+iy) = \lim_{y \rightarrow 0} \frac{0}{y^2} + 2i = 2i \end{array} \right.$$

$$\left\{ \begin{array}{l} 1+2i \neq 2i \end{array} \right.$$

$$1+2i \neq 2i$$

$$2.3 \ #2 \quad \text{Let } \lambda(z) = \frac{f(z) - f(z_0)}{z - z_0} - f'(z_0). \quad (\text{for } z \neq z_0)$$

$$\text{then } \lim_{z \rightarrow z_0} \frac{f(z) - f(z_0)}{z - z_0} = f'(z_0)$$

$$\Rightarrow \left\{ \begin{array}{l} \lim_{z \rightarrow z_0} \lambda(z) = \lim_{z \rightarrow z_0} \left[\frac{f(z) - f(z_0)}{z - z_0} - f'(z_0) \right] = 0 \end{array} \right.$$

$$\text{and } f(z) - f(z_0) + f'(z_0)(z - z_0) + \lambda(z)(z - z_0)$$

$$\begin{aligned}
 & \text{# 4(b)} \quad \lim_{\Delta z \rightarrow 0} \frac{\operatorname{Im}(z + \Delta z) - \operatorname{Im}(z)}{\Delta z} \\
 &= \lim_{\Delta x + i\Delta y \rightarrow 0} \frac{(y + \Delta y) - y}{\Delta x + i\Delta y} \\
 &= \lim_{\Delta x + i\Delta y \rightarrow 0} \frac{\Delta y}{\Delta x + i\Delta y} \\
 &= \begin{cases} i & \text{if } \Delta x = 0, \Delta y \rightarrow 0 \\ 0 & \text{if } \Delta y = 0, \Delta x \rightarrow 0 \end{cases} \quad \therefore \text{limit does not exist.}
 \end{aligned}$$

$$\#7e) \quad f'(z) = 6i \cdot 4(z^3 - 1)^3 \cdot 3z^2 (z^2 + iz)^{100} \\ + 6i(z^3 - 1)^4 \cdot 100(z^2 + iz)^{99}(2z + i)$$

$$\#(e) \quad f = (x^2+y^2+y-2) + iX$$

$$\lim_{\Delta z \rightarrow 0} \frac{f(z + \Delta z) - f(z)}{\Delta z}$$

$$= \lim_{\Delta x + i \Delta y \rightarrow 0} \frac{((x + \Delta x)^2 + (y + \Delta y)^2 + (y + \Delta y) - 2) + i(x + \Delta x)}{\Delta x + i \Delta y}$$

$$= \lim_{\Delta x + i \Delta y \rightarrow 0} \frac{(2x\Delta x + \Delta x^2 + 2y\Delta y + \Delta y^2 + \Delta y) + i \Delta x}{\Delta x + i \Delta y}$$

$$= \begin{cases} 2x+i & \text{if } \Delta y = 0, \Delta x \rightarrow 0 \\ (2y+1)i & \text{if } \Delta x = 0, \Delta y \geq 0 \end{cases}$$



these are never equal. So $f(z)$ nowhere differentiable, so nowhere analytic.