Math 253 HW1 2017 T1

You can draw some of these figures on the HW sheet itself. If you need more space, attach any additional written work.

**Q1 Parallelograms in** \( \mathbb{R}^3 \). (a) APEX 10.1 #8. (b) Also sketch and carefully label this object on a “cavalier oblique axis” (like the ones we use in lectures).

(c) Where is the center of the shape?

**Q2 Vector arithmetic.** [From APEX 10.2 #12.] Sketch the vectors \( \vec{u}, \vec{v}, \vec{u} + \vec{v}, \vec{u} - \vec{v} \) and \( 2\vec{u} - \vec{v} \) on the same axes.
Q3 Romanticizing rejection. “There’s nothing romantic about rejection. It’s horrible.” – Marlon James

(a) In terms of the dot product, what is the vector projection of a vector \( \vec{a} \) onto a vector \( \vec{v} \)?

\[
\text{proj}_{\vec{v}} \vec{a} = \]

Now define the “vector rejection” of \( \vec{a} \) onto \( \vec{v} \) by

\[
\vec{w} = \vec{a} - \text{proj}_{\vec{v}} \vec{a} =
\]

(b) Sketch and label \( \text{proj}_{\vec{v}} \vec{a} \) and \( \vec{w} \) on the following figure:

(c) Show (prove) that \( \vec{w} \) is orthogonal to \( \vec{v} \).

(c) In two dimensions, now suppose \( \vec{v} = \left( \frac{1}{3} \right) \). Consider a triangle formed by the points \( A = (10, 20) \), \( B = (20, 10) \) and \( C = (0, 10) \). By considering each point as a vector \( a \) from the origin, compute the vector rejection onto \( \vec{v} \). Plot the resulting three points.
(d) Can you make a conjecture about what the vector rejection onto $v$ does to any point in $\mathbb{R}^2$?

(e) Try computing $\vec{z} = \vec{a} - 2\text{proj}_v \vec{a}$ for each of the vertices of the triangle (again, taking these as $\vec{a}$). Plot those points. What does this new operation do to the triangle? Can you think of a good name for it?
Q4 Naughts and Crosses. [Adapted from the Stewart textbook.]

(a) Let $\vec{u} = (1, 2, 1)$. Find all vectors $\vec{v}$ such that

$$\vec{u} \times \vec{v} = (3, 1, -5).$$

(b) Of all the answers to (a), which one has no component in the $i$ direction?

(c) Are there any vectors $\vec{w}$ such that $\vec{u} \times \vec{w} = (3, 1, 5)$? Justify your answer.