Num Soln of DEs: 08b: Implicit-Explicit (IMEX) time-stepping

Splittings

Treating different terms/dimensions/waves/etc in a PDE in different ways is a big area. E.g., number of hits on Google for “Operator Splitting” or “Strang Splitting”. Here’s a few examples for time-dependent problems.

IMEX: implicit/explicit methods

Example: Kuramoto–Sivashinsky:
\[ u_t = -u_{xx} - u_{xxxx} - (u^2/2)_x \]
Or more generally:
\[ u_t = Lu + N(u), \]
Here \( L \) linear and \( N \) nonlinear operators. Simplest idea: forward Euler for \( N \) and backward Euler \( L \): IMEX Euler. [demo_08_kuramoto_sivashinsky.m]

Higher-order accuracy

For higher-order accuracy, see:


I like the “SBDF” semi-implicit BDF schemes from this last reference, particularly for reaction-diffusion problems. E.g., SBDF-2:
\[ u^{n+1} = 4/3u^n - 1/3u^{n-1} + 2k/3Lu^{n+1} + 4k/3N(u^n) - 2k/3N(u^{n-1}). \]

Exponential Time Differencing

ETDRK treats \( L \) part exactly and uses Runge–Kutta for \( N \):


Example: Korteweg–de Vries:
\[ u_t + uu_x + u_{xxx} = 0. \]

Note solitons pass through each other with no lasting effect. Numerical computations were crucial in this discovery (for example, Fornberg–Whitham paper from 1970’s).

[demo_08_etd_kdv.m] from Kassam–Trefethen. (We will discuss “spectral” spatial discretizations later).