Num Soln of DEs: 08b: Implicit-Explicit (IMEX) time-stepping

Splittings

Treating different terms/dimensions/waves/etc in a PDE in different ways is a big area. E.g., number of hits on Google for "Operator Splitting" or "Strang Splitting". Here's a few examples for time-dependent problems.

IMEX: implicit/explicit methods

Example: Kuramoto–Sivashinsky:

$$u_t = -u_{xx} - u_{xxxx} - (u^2/2)_x$$

Or more generally:

$$u_t = Lu + N(u),$$

Here L linear and N nonlinear operators. Simplest idea: forward Euler for N and backward Euler L: IMEX Euler. [demo_08_kuramoto_sivashinsky.m]

Higher-order accuracy

For higher-order accuracy, see:

- [Ascher–Ruuth–Spiteri, APNUM 1997, Implicit-explicit Runge-Kutta methods for time-dependent partial differential equations]
- [Ascher–Ruuth–Wetton, SINUM, 1995, Implicit-explicit methods for time-dependent PDE's]

I like the "SBDF" semi-implicit BDF schemes from this last reference, particularly for reaction-diffusion problems. E.g., SBDF-2:

$$u^{n+1} = 4/3u^n - 1/3u^{n-1} + 2k/3Lu^{n+1} + 4k/3N(u^n) - 2k/3N(u^{n-1}).$$

Exponential Time Differencing

ETDRK treats L part exactly and uses Runge–Kutta for N:

[Cox–Matthews, JCP 2002, "Exponential Time Differencing for Stiff Systems"]

[Kassam–Trefethen, SISC 2005, "Fourth-Order Time-Stepping for Stiff PDEs"]

Example: Korteweg–de Vries:

$$u_t + uu_x + u_{xxx} = 0.$$

Note solitons pass through each other with no lasting effect. Numerical computations were crucial in this discovery (for example, Fornberg–Whitham paper from 1970's).

[demo_08_etd_kdv.m] from Kassam–Trefethen. (We will discuss "spectral" spatial discretizations later).