Here is a list of possible project topics, most of which came up naturally during the course. They are just meant to give you a starting point. You can add or remove subjects from each one of them. Even better, I encourage you to come up with your own topic. Almost anything goes, but do run it by me first.

The presentation should be 30 minutes long (plus 5-10 minutes for questions and discussion). This is not much time so make sure you prepare your presentation carefully. Clarity is important. Examples are great!

1. Explain in more detail the analogy between the triangulate category structure on the homotopy category of topological spaces and the triangulate category structure on the homotopy category of an abstract additive category $\mathcal{C}$. For example, explain how the cone construction are related. What about spectra?

2. Discuss what happens if you replace Grassmannians $\text{Gr}(k, N)$ with more general $m$-step flag varieties. In particular, explain how you end up going from $\mathfrak{sl}_2$ actions to $\mathfrak{sl}_m$ actions.

   In the story above explain how the equivalences $T$ in the $\mathfrak{sl}_2$ case generalize to give you a braid group action (on $m$ strands) in the $\mathfrak{sl}_m$ case.

3. What changes if you replace the category of constructible sheaves with the category of coherent sheaves? Discuss some of the basics – for instance, how this means working with $\mathcal{O}_X$-modules and the definition of $f^*$. Why is $f^*$ for coherent sheaves more “complicated” than $f^{-1}$ for constructible sheaves? Is $f^*$ exact? Work out some examples.

4. Explain roughly what is a (filtered) $\mathcal{D}_X$-module. If $M$ is a filtered $\mathcal{D}_X$-module explain why the associated graded $gr(M)$ is a $\mathcal{O}_{T^*X}$-module. This explains why the categorical $\mathfrak{sl}_2$ action on $\bigoplus_k \mathcal{D} \text{Con}(\text{Gr}(k, N))$ implies (modulo some technical issues) a categorical $\mathfrak{sl}_2$ action on $\bigoplus_k \mathcal{D} \text{Coh}(T^*\text{Gr}(k, N))$.

5. Explain what are fine sheaves and how they can be used to compute cohomology (discuss soft sheaves?). In particular, show why this implies that sheaf cohomology $H^*(X, \mathbb{C}_X)$ of the constant sheaf on a manifold $X$ is the same as DeRham cohomology $H^*_\text{dR}(X)$.

   Explain why the sheaf cohomology $H^i(X, \mathbb{C}_X)$ is isomorphic to singular cohomology $H^i_{\text{sing}}(X, \mathbb{C})$.

6. Let $G$ be a group. Explain what is a principal $G$-bundle. Explain why principal $G$-bundles are parametrized by the Cech cohomology $\check{H}^1(X, \mathcal{G})$ (more precisely, the 1-cycles correspond to principal $G$-bundles and modding out by the image of
0-cycles identifies isomorphic $G$-bundles). To clarify what this means concentrate on the case $G = \mathbb{C}^*$ where

- $G = \mathbb{C}^*$ is given the discrete topology (so you get rank one complex local systems)
- $G = \mathbb{R}^*$ is given the smooth topology (so you get smooth line bundles)
- $G = \mathbb{C}^*$ is given the complex topology (so you get complex line bundles)

In the last case explain the definition and relevance of the short exact sequence

$$0 \to \mathbb{Z} \to \mathcal{O}_X \to \mathcal{O}_X^* \to 0$$

and discuss what you get by considering the associated long exact sequence.