1. TRUE or FALSE:

(a) If $A$ is an $n \times n$ matrix with nonzero determinant and $AB = AC$ then $B = C$.

(b) A square matrix with zero diagonal entries is never invertible.

(c) A linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^n$ is one-to-one if and only if its standard matrix has nonzero determinant.

(d) Every spanning subset of $\mathbb{R}^4$ contains a basis for $\mathbb{R}^4$.

(e) A linearly independent subset of $\mathbb{R}^n$ has at most $n$ elements.

(f) Every subspace of $\mathbb{R}^3$ contains infinitely many vectors.

(g) The system of linear equations $Ax = b$ has a solution if and only if $b$ is in the column space of $A$.

2. Indicate if each of the following is a linear subspace:

(a) The set of all vectors parallel to a fixed vector $v$.

(b) All the vectors \( \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \) with $x_1 + x_3 = 1$.

(c) The intersection of two subspaces of $\mathbb{R}^n$.

(d) The set of vectors in $\mathbb{R}^3$ with two equal components.
3. Determine if each of the following matrices is invertible? If not, explain why. If so, compute its inverse.

\[
\begin{pmatrix}
1 & 0 & -1 & 2 \\
0 & 3 & 0 & -4 \\
3 & -2 & -2 & 8 \\
1 & 1 & 0 & 0
\end{pmatrix}
\begin{pmatrix}
1 & 1 & 2 \\
3 & 0 & 1 \\
-1 & 1 & 1
\end{pmatrix}
\]

4. Compute the determinant of the following matrices:

\[
\begin{pmatrix}
0 & 2 & 3 \\
-1 & -1 & 4 \\
2 & -2 & 2
\end{pmatrix}
\begin{pmatrix}
0 & 0 & 5 & 5 \\
0 & 2 & 6 & 12 \\
1 & 4 & 7 & 12 \\
2 & 8 & 14 & 15
\end{pmatrix}
\]

5. Suppose \( A = (a_1 \cdots a_n) \) is an \( n \times n \) matrix with columns \( a_1, \ldots, a_n \). For each of the following statements, indicate whether they are true or false and justify your answer.

(a) If \( \det(A) = 0 \), then the set \( \{a_1, \ldots, a_n\} \) is linearly dependent.

(b) If \( \det(A) = 0 \), then \( a_n \) is a linear combination of \( \{a_1, \ldots, a_{n-1}\} \).

(c) If \( \{a_1, \ldots, a_{n-1}\} \) is linearly independent and \( \det(A) = 0 \), then \( a_n \) is a linear combination of \( \{a_1, \ldots, a_{n-1}\} \).

(d) If the system of linear equations \( Ax = b \) has a solution, then the determinant of the matrix \( B = (a_1 \cdots a_{n-1} \ b) \), obtained from replacing the last column of \( A \) by \( b \), is zero.

(e) The exists a vector \( c \in \mathbb{R}^n \) which is not in the span of \( \{a_1, \ldots, a_{n-1}\} \).

(f) If \( \{a_1, \ldots, a_{n-1}\} \) is linearly independent then there exists a vector \( c \) such that the determinant of the matrix \( C = (a_1 \cdots a_{n-1} \ c) \), obtained from replacing the last column of \( A \) by \( c \), has nonzero determinant.

6. Given a set of vectors \( \{a_1, \ldots, a_n\} \) in \( \mathbb{R}^n \), if \( A \) is the matrix \( A = (a_1 \cdots a_n) \), we write

\[
\det(a_1, \ldots, a_n) = \det(A)
\]

(a) Suppose that \( \det(u, v, w, z) = 2 \) for a set of vectors \( \{u, v, w, z\} \) in \( \mathbb{R}^4 \). Find:

\[
\det(w + 2v, v, z, 3u)
\]