1. section 1.3, #36
If you write out the determinant and expand you find that you get a linear equation in $x, y, z$. So the expression defines a plane. To see that it’s is the plane through the points $A, B$ and $C$ notice that if you plug in $(a_1, a_2, a_3)$ then the first row becomes zero so the determinant is zero. Therefore the plane contains the point $A$. Similarly with $B$ and $C$.

2. section 1.3, #38
The first equality is obtained by subtracting 4 times the first row from the second row.
The second equality is obtained by subtracting 7 times the first row from the third row.
The third equality follows since the first column has all zeros except in the top spot. If you’d like to use the definition from class you can subtract appropriate multiplies of the first column from the second and third columns so that along the top row you get $(1, 0, 0)$ and then the claim is obvious.
The fourth equality follows from the definition of the determinant for 2 by 2 matrices.

3. section 1.4, #8
8a) In cylindrical coordinates $x = r \cos(\theta)$ and $z = z$ so the plane is $z = r \cos(\theta)$.
8b) In spherical coordinates $x = \rho \sin \phi \cos \theta$ and $z = \rho \cos \phi$ so the plane is $\rho \sin \phi \cos \theta = \rho \cos \phi$.

4. section 1.4, #10
Remember that $\rho$ denotes the distance from the origin. Thus $\rho = 2f(\theta, \phi)$ denotes the same surface but each point is twice the distance from the origin (a dilation). Similarly $\rho = -f(\theta, \phi)$ denotes the surface obtained by flipping through the origin (taking the mirror image through the point $(0, 0, 0)$).
Combining these two observations we get that $\rho = -2f(\theta, \phi)$ is the surface obtained from $\rho = f(\theta, \rho)$ by dilating with respect to the origin by a factor of 2 and the flipping through the origin.

5. section 1.5, #8
Without showing the details we have:

$$AB = \begin{pmatrix} 3 & 1 & -3 \\ 5 & -1 & 1 \\ 1 & 1 & -1 \end{pmatrix}$$
while $\det(A) = 3 \cdot 2 - 0 + 1 \cdot (-2) = 4$, $\det(B) = 1 \cdot (-1) - 0 + (-1)(2) = -3$ and $\det(AB) = -12$. Finally,

$$\det(A + B) = \det \begin{pmatrix} 4 & 0 & 0 \\ 3 & 2 & 0 \\ 1 & 1 & 1 \end{pmatrix} = 8.$$ 

6. section 1.5, #12

It is not true that $\det(A + B) = \det(A) + \det(B)$. For a counterexample take the matrices $A$ and $B$ from the previous exercise (section 1.5 #8). Can you find an easier counterexample?

7. section 2.1, #12

The level surface is $4x^2 + y^2 + 9z^2 = c$. This is similar to $x^2 + y^2 + z^2 = c$ which would describe a sphere. In fact it is the dilation of a sphere, namely an ellipsoid – if you can’t visualize this think of the circle and how an ellipse is a dilation of the circle).

We can take the plane $y = 0$ and slice the graph to get $w = 4x^2 + 9z^2$ which looks like a bowl opening up. Similarly, if we slice it with $x = 0$ we get $w = y^2 + 9z^2$ which is another bowl opening up.

8. section 2.1 #32

The level surface is $\frac{2xy}{x^2 + y^2} = k$ or equivalently $2xy = k(x^2 + y^2)$. Changing to polar coordinates this is

$$2r \cos \theta \cdot r \sin \theta = kr^2(\cos^2 \theta + \sin^2 \theta),$$

or equivalently $2 \cos \theta \sin \theta = k$. One can visualize this better if you recall that $2 \cos \theta \sin \theta = \sin(2\theta)$ so that you get

$$\sin(2\theta) = k.$$

This means that the slice is empty unless $-1 \leq k \leq 1$ in which case it looks like two lines through the origin (corresponding to $\theta = \text{constant}$). The angle at which they meet varies with $k$ and is zero when $k = \pm 1$ and 90 degrees when $k = 0$.

9. section 2.2, #10b

First you have to decide whether you think the limit should exist or not. In this case it looks like the denominator $x^4 + y^4$ should tend to zero much faster than the numerator so you might guess the limit should not exist.

Let’s try approaching the origin along the $x = y$ direction (ie. along points of the form $(t, t)$). Then the limit should equal to

$$\lim_{t \to 0} \frac{\cos(t) - 1 - (t^2/2)}{2t^4}.$$
This is a one variable limit which you can calculate using your favourite technique from simple variable calculus. Since both the numerator and denominator tend to zero it is probably easiest to do this using L’hopital’s theorem. Differentiating the top and bottom gives the limit

\[ \lim_{t \to 0} \frac{-\sin(t) - t}{8t^3} \]

We can use L’hopital’s theorem again to get

\[ \lim_{t \to 0} \frac{-\cos(t) - 1}{24t^2} \]

and now the numerator tends to \(-2\) while the denominator tends to 0 so the limit does not exist!

10. section 2.2, #18

Let’s use spherical coordinates (since we did not discuss \(\epsilon, \delta\) proofs. Then

\[ xyz = (\rho \sin \phi \cos \theta)(\rho \sin \phi \sin \theta)(\rho \cos \phi) = \rho^3 (\sin^2 \phi \cos \phi \cos \theta \sin \theta) \]

and

\[ x^2 + y^2 + z^2 = \rho^2 \]

The limit \((x, y, z) \to (0, 0, 0)\) is the same as \(\rho \to 0\) so we get the limit

\[ \lim_{\rho \to 0} \frac{\rho^3 (\sin^2 \phi \cos \phi \cos \theta \sin \theta)}{\rho^2} = \lim_{\rho \to 0} \frac{\rho (\sin^2 \phi \cos \phi \cos \theta \sin \theta)}{\rho} = 0. \]