Instructions: You have 2 hours to complete this exam. You should work alone, without access to the textbook or class notes. You may not use a calculator. Do not discuss this exam with anyone except your instructor.

This exam consists of 6 questions. You must show your work to receive full credit. Be sure to indicate your final answer clearly for each question. Pledge your exam when finished, and include your name and section number on the front of the exam. The exam is due by Wednesday, 4 p.m. Good luck!

1. Find and classify all the critical points of 
   \[ f(x, y) = \frac{1}{2}x^2 - xy + \frac{1}{3}y^3. \]

2. Let \( g(x, y) = 2e^{-x} \cos y. \)
   (a) Find the quadratic Taylor polynomial for \( g(x, y) \) around the point \( (0, 0) \).
   (b) Use your answer in part (a) to estimate \( 2e^{-0.2} \cos 0.4 \).

3. A tank is in the shape of a half-cylinder of radius 2 and height 3. It is situated in \( \mathbb{R}^3 \), given by the inequalities \( \sqrt{x^2 + y^2} \leq 2, y \geq 0, \) and \( 0 \leq z \leq 3 \). The temperature at the point \((x, y, z)\) is given by 
   \[ T(x, y, z) = 2yz^2 \sqrt{x^2 + y^2} \degree \text{C}. \]
   Find the average temperature in the tank.

4. Let \( T \) be the triangle with vertices \((0, 0), (1, 1)\) and \((0, 1)\) and let \( f(x, y) = x \sin(y^3) \).
   (a) Find the correct limits of integration to set up \( \iint_T f(x, y) \, dA \) as a double integral 
       \( \iint f(x, y) \, dx \, dy \).
   (b) Find the correct limits of integration to set up \( \iint_T f(x, y) \, dA \) as a double integral 
       \( \iint f(x, y) \, dy \, dx \).
   (c) Compute \( \iint_T f(x, y) \, dA \).

5. Find the maximum and minimum values obtained by \( f(x, y) = x + y^2 \) on the ellipse \( x^2 + 3y^2 \leq 9 \).

6. The region \( S \) is cut from a solid ball of radius 1 centered at the origin. \( S \) is the region cut by the inequalities \( z \geq 0 \) and \( y \geq x \). (\( S \) is one-quarter of the entire ball, and contains the point \((0, 1, 0)\).)
   The mass density of \( S \) at a point \((x, y, z)\) is given by the function \( \delta(x, y, z) = 30z^2 \text{ kg/m}^3 \).
   (a) Find the total mass of \( S \).
   (b) Find the average mass density of \( S \).