1. section 4.1, #18

\[ \frac{y_2(t)}{y_1(t)} = \tan(3t) \] which is not a constant. Hence \( y_1 \) and \( y_2 \) are linearly independent. Also, the Wronskian is

\[
\det \begin{pmatrix}
\cos(3t) & \sin(3t) \\
-3\sin(3t) & 3\cos(3t)
\end{pmatrix} = 3(\cos^2(3t) + \sin^2(3t)) = 3
\]

2. section 4.1, #22

It’s easy to check \( y_1 \) and \( y_2 \) are solutions. Also, they are linearly independent since \( \frac{y_2}{y_1} = e^{-4t} \) which is not constant. Hence the general solution is \( y(t) = Ae^t + Be^{-3t} \). So \( y(0) = 1 \) implies \( A + B = 1 \) while \( y'(0) = -2 \) implies \( A - 3B = -2 \). Subtracting them gives \( -4B = 3 \) so \( B = \frac{3}{4} \) and \( A = \frac{1}{4} \).

3. section 4.3, #4

The characteristic polynomial is \( \lambda^2 + \lambda - 12 = (\lambda + 4)(\lambda - 3) \). Hence the general solution is \( Ae^{3t} + Be^{-4t} \).

4. section 4.3, #14

The characteristic polynomial is \( \lambda^2 + 2\lambda + 3 \) which has roots \( -1 \pm \frac{\sqrt{-8}}{2} \). Hence the general solution is spanned by complex functions \( e^{-t} \cdot e^{it\sqrt{2}} \) and \( e^{-t} \cdot e^{-it\sqrt{2}} \). Taking linear combination of these (changing basis) we get two real solutions \( e^{-t}\cos(t\sqrt{2}) \) and \( e^{-t}\sin(t\sqrt{2}) \). So the general solutions is \( e^{-t}(A\cos(t\sqrt{2}) + B\sin(t\sqrt{2})) \).

5. section 4.3, #20

The characteristic polynomial is \( 4\lambda^2 + 12\lambda + 9 = (2\lambda + 3)^2 \). Hence the general solution is \( Ae^{-3t/2} + Be^{-3t/2} \).

6. section 4.3, #32

The characteristic polynomial is \( \lambda^2 - 4\lambda - 5 = (\lambda + 1)(\lambda - 5) \). Hence the general solution is \( Ae^{-t} + Be^{5t} \).

7. section 4.3, #37

If \( \lambda_1 \) and \( \lambda_2 \) are the roots then we can factor as

\[
\lambda^2 + p\lambda + q = (\lambda - \lambda_1)(\lambda - \lambda_2)
\]

Expanding the right side we get \( \lambda^2 - \lambda(\lambda_1 + \lambda_2) + \lambda_1\lambda_2 \). Since two polynomials are equal only if their coefficients are equal we compare coefficients with \( \lambda^2 + p\lambda + q \) to find that \( -(\lambda_1 + \lambda_2) = p \) and \( \lambda_1\lambda_2 = q \).
The amplitude is \( A = \sqrt{a^2 + b^2} = \sqrt{3e^{-2t/4} + e^{-2t/4}} = 2e^{-t/4} \). Also the fundamental frequency is \( \omega = 4 \) while the phase \( \phi \) satisfies \[ \tan(\phi) = b/a = -1/\sqrt{3} \]

which means \( \phi = -\pi/6 \) (−30 degrees). Hence \( y \) takes the form
\[
y = 2e^{-t/4} \cos(4t + \pi/6)
\]

Its graphs oscillates and decays to zero as \( t \to \infty \). On the other hand, the graphs of \( \pm 2e^{-t/4} \) don’t oscillate and just tend directly to zero as \( t \to \infty \) while passing through \( (t, y) = (0, \pm 2) \).