1. section 2.9, #8

Equilibrium points: \(y_0(t) = 0\) and \(y_1(t) = 2\) are equilibrium solutions. The solution \(y_0\) is stable and \(y_1\) is unstable.

2. section 2.9, #10

The equilibrium solutions are \(y_0(t) = -2\), \(y_1(t) = -\frac{2}{3}\), \(y_2(t) = 1\) and \(y_3(t) = 2\). The stable solutions are \(y_0\) and \(y_2\).

3. section 2.9, #20

The equilibrium points are \(y_0 = 3\), \(y_1 = 1\) and \(y_2 = 3\). \(y_1\) is stable.

4. section 2.9, #28

The equilibrium points are \(x_0 = 2\), \(x_1 = 0\) and \(x_2 = 1\). \(x_1\) is stable.

5. section 3.1, #4

\(P(t) = Ce^{rt}\) is our model. \(P(0) = 1000\) also \(P(10) = 2 \times 1000 = 2000 = 1000e^{10r}\)
so, \(r = \frac{\ln 2}{10}\).
\(P(t_0) = 10000\) then \(1000e^{\frac{\ln 2}{10}t_0} = 10000\)
So \(t_0 = 10 \times \frac{\ln 10}{\ln 2}\).

6. section 3.1 #10

Find the maximum rate of growth for the logistic equation. The maximum rate of growth is the maximum of \(\frac{dP}{dt}\). To do this find the zeros of the second derivative which is:
\[
\frac{d^2P}{dt^2} = rP' - r\frac{3kPP'}{k + P} = rP'(1 - \frac{2k}{k + P})
\]
This is zero when \(P = \frac{k}{2}\) or when the population is half of the maximum. This is a maximum because of the sign of \(\frac{d^2P}{dt^2}\) at \(P = \frac{k}{2} - \epsilon > 0\) and \(P = \frac{k}{2} + \epsilon < 0\).

7. section 3.1 #12

\(P(t) = \frac{Kce^{rt}}{1 + ce^{rt}}\)
\(P(0) = 1000\), \(P(10) = 2 \times 1000 = 2000\), \(K = 20,000\)
Find \(P(25)\):
\[
P(0) = 1000 = \frac{20,000e^0}{1 + ce^0} \quad \quad \quad \quad c = \frac{1}{19}
\]
\[
P(10) = 2000 = \frac{20,000 + ce^{10}}{1 + ce^{10}} \quad \quad \quad 2000 = e^{10r} \frac{1}{19} (20,000 - 2000) \text{ Meaning, } r = \frac{1}{19} \ln(\frac{10}{19})
\]
\[
P(25) = \frac{20,000e^{25}}{1 + ce^{25}} \text{ is roughly 5083.75}
\]
8. section 3.3 #4

$M' = .0625M + D$ so

$(e^{-0.0625t} M)' = e^{-0.0625t} D$ Which means

$e^{-0.0625t} M = -\frac{1}{0.0625} e^{-0.0625t} D + C$ or $M(t) = -\frac{1}{0.0625} D + C(e^{0.0625t})$.

Now $M(0) = 0$ so $C = \frac{1}{0.0625} D$. We also know $M(18) = 50,000$

$C(e^{0.0625*18}) - C = 50,000$ and $C = \frac{50000}{e^{0.0625}}$, $D = .0625 \cdot \frac{50000}{e^{0.0625}}$

9. section 4.1 #10

$k \cdot \frac{3}{4} = 5 \cdot 9.8$

$5y'' + \frac{20 + 9.8}{3} y = 0$ and $y(0) = .36$, $y'(0) = -.45$

10. section 4.1 #14 $(C_1 \cos(2t) + C_2 \sin(2t))'' + 4(C_1 \cos(2t) + C_2 \sin(2t)) = -4C_1 \cos(2t) + -4C_2 \sin(2t) + 4(C_1 \cos(2t) + C_2 \sin(2t)) = 0$