Math 211  
First Midterm  
September 25, 2001

Make sure to show your work and justify your arguments.

**Calculator policy:** You may use calculators to evaluate standard functions on floating point numbers (like \( \sqrt{3.12} \), \( \ln(35/7) \), or \( \sin(\pi/17) \)). You may not use symbolic operations, numerical integration, or any graphing functions.

1) Consider the differential equation 
\[ y'' - y' + y = -\cos t. \]

a) Is \( y(t) = \sin t \) a solution? (7%)  
b) Is \( y(t) = 2\sin t \) a solution? (7%)

2) Consider the differential equation 
\[ y'(1 - t) = y. \]

a) Give the general solution. (9%)  
b) Find a particular solution with \( y(2) = 1 \) and give its interval of existence. (6%)

3) Find a solution to the initial value problem 
\[ y' = \frac{2}{t}y + \frac{\cos t}{t^2}, \quad y(\pi) = 0. \] (14%)

4) Consider a pond with 1000 cubic meters of water. There is a stream flowing out from the pond, at a rate of 10 cubic meters a day. Nearby is a field which is regularly irrigated and fertilized. Each day, 10 cubic meters of water from the field enters the pond, and this is contaminated with 3 kilograms of ammonium nitrate per cubic meter.

Write down a differential equation for the amount of ammonium nitrate in the pond. Assume the ammonium nitrate is perfectly mixed and ignore the effect of rain and evaporation. **Do not attempt to solve this differential equation!** (12%)  

**EXAM CONTINUES ON BACK PAGE**
5) Analyze the stability of the differential equation

\[ \frac{dy}{dt} = 3 + 2y - y^2. \]

In particular:

a) Sketch the graph of \( f(y) = 3 + 2y - y^2 \) and identify the equilibrium points. (7%)

b) Draw the phase line and analyze the stability near each equilibrium point. (7%)

c) Consider the solution \( y(t) \) with initial value \( y(0) = 0 \). Describe its behavior as \( t \to \infty \). Does it approach any of the equilibrium solutions? (7%)

6) A ball with mass 1/4 kilogram is thrown upward with a high initial velocity. Assume that the air resistance is given by

\[ R = -0.05v|v|, \]

where \( v \) is the velocity, and the gravitational constant is

\[ g = 9.8 \text{ meters per second}^2. \]

Compute the terminal velocity of the ball. (10%)

7) Consider the differential equation

\[ y' = \frac{e^t (4 - y^2)}{4e^t - 2}. \]

a) Show that \( y(t) = 2 - 2e^{-t} \) satisfies the differential equation. (7%)

b) Consider the solution \( \tilde{y}(t) \) of the initial value problem \( y(0) = 1 \). Show that

\[ \lim_{t \to \infty} \tilde{y}(t) = 2. \] (7%)

**Hint:** It is not necessary to compute \( \tilde{y} \).