1. section 10.7, #4
The sequence \( \left\{ \frac{n^3}{n^2 + 2} \right\} \) is monotonically decreasing with limit 0, so the given series meets both criteria of the alternating series test. Therefore this series converges.

2. section 10.7, #12
The series diverges by the nth-term test for divergence because \((\frac{2n}{10})^{n+1}\) goes to \(+\infty\) as \(n\) goes to \(+\infty\).

3. section 10.7, #26
The ratio test yields 
\[
\rho = \lim_{n \to \infty} \frac{3^{n+1}n!n}{3^n(n+1)!} = \lim_{n \to \infty} \frac{3n}{(n+1)^2} = 0.
\]
Therefore the given series converges absolutely.

4. section 10.7, #32
Because \(\lim_{n \to \infty} \frac{2^{3n}}{7^n} = \frac{8^n}{7^n} = \lim_{n \to \infty} (\frac{8}{7})^n = +\infty\), the series diverges by the nth-term test for divergence.

5. section 10.7, #40
The ratio test yields 
\[
\rho = \lim_{n \to \infty} \frac{1 \cdot 3 \cdot 5 \cdots (2n - 1) \cdot (2n + 1)}{1 \cdot 3 \cdot 5 \cdots (2n - 1) \cdot 1 \cdot 4 \cdot 7 \cdots (3n - 2) \cdot (3n + 1)} = \lim_{n \to \infty} \frac{2n + 1}{3n + 1} = \frac{2}{3}.
\]
Because \(\rho < 1\), the original series converges absolutely.

6. section 10.7, #52
The condition \(\frac{1}{(2n)!} < 0.000005\) leads to \(4 < n < 5\), so the sum of the terms through \(n = 4\) will provide five-places accuracy. The sum of the first terms of the series is 
\[
\sum_{n=0}^{4} \frac{(-1)^n}{(2n)!} = \frac{4357}{8064} \approx 0.5403025
\]
so to five places, the sum of the infinite series is 0.54030.

7. section 10.7, #60
Part(a), The ratio test yields 
\[
\rho = \lim_{n \to \infty} \frac{(n + 1)|r|^{n+1}}{n|r|^n} = \lim_{n \to \infty} \frac{(n + 1)|r|}{n} = |r| < 1.
\]
Therefore the series in question converges.
Part(b), Let $S$ denote the sum of the series in Part(a). Then

$$(1 - r)S = \lim_{k \to \infty} \sum_{n=0}^{k} (nr^n - nr^{n+1})$$

$$= \lim_{k \to \infty} (r - r^2 + 2r^2 - 2r^3 + 3r^3 - 3r^4 + \cdots + (k - 1)r^{k-1} - (k - 1)r^k + kr^k - kr^{k+1})$$

$$= \lim_{k \to \infty} (1 + r + r^2 + r^3 + \cdots + r^k - kr^{k+1} - 1)$$

$$= \lim_{k \to \infty} \left( \frac{1 - r^{k+1}}{1 - r} - kr^{k+1} - 1 \right)$$

$$= \frac{r}{1 - r}.$$ 

Thus $\sum_{n=0}^{\infty} nr^n = S = \frac{r}{(1 - r)r}$.

8. section 10.8, #8

The ratio test yields

$$\lim_{n \to \infty} \frac{(2n + 1)^{1/2}4^{n+1}|x|^{n+1}}{(2n + 3)^{1/2}4^n|x|^n} = 4|x|.$$ 

So the series converges if $-\frac{1}{4} < x < \frac{1}{4}$. When $x = 1/4$, the series converges by the alternating series test. When $x = -1/4$, the series diverges by limit-comparison test with the p-series for which $p = 1/2$, hence $(-1/4, 1/4]$.

9. section 10.8, #26

The ratio test yields

$$\lim_{n \to \infty} \frac{n \cdot 10^n \cdot |x - 2|^{n+1}}{(n + 1) \cdot 10^{n+1} \cdot |x - 2|^n} = \frac{|x - 2|}{10},$$

and therefore this series converges if $-10 < x - 2 < 10$, that is, if $-8 < x < 12$. If $x = 12$ it converges by the alternating series test; if $x = -8$ it diverges because it becomes harmonic series, hence $(-8, 12]$.

10. section 10.8, #28

Notice that the given series is actually geometric with first term 1 and ratio $r = \frac{x^2 + 1}{5}$. Hence it converges if $-1 < r < 1$, that is, $-2 < x < 2$. So its interval of convergence is $(-2, 2)$ and its sum is $\frac{5}{4 - x^2}$. 