1. section 10.3, #2
This is a geometric series with $a = 1$ and $r = \frac{1}{e}$. Since $|r| < 1$ this series converges and the sum is
$$\sum_{n=0}^{\infty} e^{-n} = \frac{1}{1 - \frac{1}{e}} = \frac{e}{e-1}$$

2. section 10.3, #6
This is a geometric series with $a = 1$ and $r = \frac{-1}{4}$. Since $|r| < 1$ this series converges and the sum is
$$\sum_{n=0}^{\infty} \left(\frac{-1}{4}\right)^n = \frac{1}{1 + \frac{1}{4}} = \frac{4}{5}$$

3. section 10.3, #14
First,
$$\sum_{n=0}^{\infty} \frac{3^n - 2^n}{4^n} = \sum_{n=0}^{\infty} \frac{3^n}{4^n} - \sum_{n=0}^{\infty} \frac{2^n}{4^n}$$
These are two geometric series, the first with $a = 1$ and $r = \frac{3}{4}$, and the second with $a = 1$ and $r = \frac{1}{2}$. In both cases $|r| < 1$ this series converges and the sum is
$$\frac{1}{1 - \frac{3}{4}} - \frac{1}{1 - \frac{1}{2}} = 4 - 2 = 2$$

4. section 10.3, #20
This series diverges by the $n$th term test, since
$$\lim_{n \to \infty} \frac{1}{1 + \left(\frac{9}{10}\right)^n} = \frac{1}{1 + 0} = 1 \neq 0$$

5. section 10.3, #22
First,
$$\sum_{n=1}^{\infty} \left(\frac{\pi}{e}\right)^n = \sum_{n=0}^{\infty} \left(\frac{\pi}{e}\right)^n - 1$$
Now, $\sum_{n=0}^{\infty} \left(\frac{\pi}{e}\right)^n$ is a geometric series with $a = 1$ and $r = \frac{\pi}{e}$, since $\pi > e$, $|r| > 1$ so this series diverges. Thus the sum $\sum_{n=1}^{\infty} \left(\frac{\pi}{e}\right)^n$ diverges.

6. section 10.3, #28
By the $n$th term test this series diverges, since
$$\lim_{n \to \infty} 2^{\frac{n}{2}} = 1$$
7. section 10.3, #42

\[
0.337733773377 \cdots = \frac{3377}{10^4} + \frac{3377}{10^8} + \frac{3377}{10^{12}} + \cdots = \frac{\frac{3377}{10000}}{1 - \frac{1}{10000}} = \frac{3377}{9999} = 0.3377
\]

8. section 10.3, #46

The given geometric series has \(a = 1 - x\) and \(r = 1 - x\). The series converges when \(|r| < 1\), i.e. \(|1 - x| < 1\) so \(0 < x < 2\). Then for such an \(x\) the sum is,

\[
\sum_{n=1}^{\infty} (x - 1)^n = \frac{x - 1}{1 - (x - 1)} = \frac{x - 1}{2 - x}
\]

9. section 10.3, #50

The method of partial fractions gives,

\[
\frac{1}{4n^2 - 1} = \frac{1}{2} \left( \frac{-1}{2n - 1} + \frac{-1}{2n + 1} \right)
\]

So the \(k\)th partial sum of the series is

\[
S_k = \sum_{n=1}^{k} \frac{1}{2} \left( \frac{-1}{2n - 1} + \frac{-1}{2n + 1} \right) = \frac{1}{2} \left( 1 - \frac{1}{3} - \frac{1}{3} + 1 - \frac{1}{5} - \frac{1}{5} + \cdots - \frac{1}{2k + 1} \right) = \frac{1}{2} \left( 1 - \frac{1}{2k + 1} \right)
\]

Thus

\[
\sum_{n=1}^{\infty} \frac{1}{4n^2 - 1} = \lim_{k \to \infty} S_k = \frac{1}{2}.
\]

10. section 10.3, #58

The method of partial fractions gives,

\[
\frac{2}{n(n+1)(n+2)} = \frac{1}{n} - \frac{2}{n+1} + \frac{1}{n+2}
\]

So the \(k\)th partial sum of the series is
\[ S_k = \sum_{n=1}^{k} \frac{2}{n(n+1)(n+2)} = \frac{1}{1} - \frac{2}{2} + \frac{3}{3} + \frac{1}{2} - \frac{2}{3} + \frac{4}{4} + \frac{1}{3} - \frac{2}{4} + \frac{5}{5} + \frac{1}{4} - \frac{2}{5} + \frac{6}{6} + \frac{1}{5} - \frac{2}{6} + \frac{7}{7} + \frac{1}{6} - \frac{2}{7} + \frac{8}{8} + \frac{1}{7} - \frac{2}{8} + \frac{9}{9} + \frac{1}{8} - \frac{2}{9} + \frac{10}{10} + \cdots + \frac{1}{k-2} - \frac{2}{k-1} + \frac{1}{k} + \frac{1}{k-1} - \frac{2}{k} + \frac{1}{k+1} + \frac{1}{k} - \frac{2}{k+1} + \frac{1}{k+2}. \]

Inspect the diagonals running from the top right to lower left. Those fractions with denominator 3 cancel one another, as do those with denominators 4, 5, 6, \ldots, k−1, k. So

\[ S_k = \frac{1}{1} - \frac{2}{2} + \frac{1}{2} + \frac{1}{k+1} - \frac{2}{k+1} + \frac{1}{k+2} = \frac{1}{2} - \frac{1}{k+1} + \frac{1}{k+2}. \]

Thus the sum

\[ \sum_{n=1}^{\infty} \frac{2}{n(n+1)(n+2)} = \lim_{k \to \infty} S_k = \frac{1}{2}. \]