Second Midterm Practice test 1

1. Does the series
\[ \sum_{n=2}^{\infty} \frac{5 \cdot 3^n}{4n+1} \]
converge or diverge? If it converges, find its sum.

2. Find the third degree Taylor formula for \( f(x) = \tan x \) at \( x = 0 \).

3. Does the series
\[ \sum_{n=1}^{\infty} \frac{5n + 19}{n^2 + 7n + 10} \]
converge or diverge?

4. Does the series
\[ \sum_{n=1}^{\infty} \frac{\cos^2(1/n)}{\sqrt{n^3 + 1}} \]
converge or diverge?

5. Does the series
\[ \sum_{n=1}^{\infty} \frac{2^{n-1}(n!)^2}{(2n)!} \]
converge absolutely, converge, or diverge?

6. Find the interval of convergence for the power series
\[ \sum_{n=0}^{\infty} \frac{(-3)^n(x - 1)^n}{\sqrt{n + 3}}. \]
Answers

1. Converges to 45/16.

2. \[ \tan x = x + \frac{1}{3}x^3 + \frac{1}{24}x^4(8 \sec^2 z \tan^3 z + 16 \sec^4 z \tan z) \]
   where \( z \) is some real number between 0 and \( x \).

3. Diverges (you could use the integral test)

4. Converges (you could use comparison tests)

5. Converges absolutely

6. \((2/3, 4/3]\)